A TREATISE

ON THE

STRENGTH OF BRIDGES AND ROOFS.

Frank W. Maher
San Francisco

PREFACE.

A treatise on the strength of

BRIDGES AND ROOFS,

WITH

PRACTICAL APPLICATIONS AND EXAMPLES,

FOR THE USE OF

ENGINEERS AND STUDENTS.

BY

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NEW YORK:
D. VAN NOSTRAND, PUBLISHER,
23 Murray St. and 27 Warren St.
1873.

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PREFACE.

In the following treatise I have applied the simpler processes of Algebra to the discussion of the subject of strains in single span trusses, and have obtained many formulæ for practical application, sufficiently elucidating this method, I hope, to render easy the determination of other formulæ adapted to any form of truss that ingenuity may suggest. Only algebraic processes have been employed, because they are simpler, more comprehensible, more practical, and more accurate in practice, than those of the higher mathematics.

Uniformly distributed loads alone have been considered, whether full or partial, since it may be possible to load each part with the load which may be brought upon any one part; and if we consider the density of the whole load to equal the maximum density at any point, any other case can only produce a less strain.

No comparison is made between the different systems which are given, many of them for the first time, since practical details of construction affect the theoretical economy, and to fully consider these would be beyond the scope of the present volume; there are many cases, however, where the practical difficulties may be ignored, since they are so nearly equal and a comparison readily made. The changes in the form of a truss caused by a want of rigidity, or temperature, will not affect the values of the formulæ. The discussion of the strains affecting drawbridges, leading directly to the subject of cantilever trusses, which includes continuous trusses, the Sedley system, and many other important forms, for which we have no practical formulæ, must, with the subject of arched trusses, be reserved for a future volume.

S. H. S.

NEW YORK, *January*, 1873.

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A TREATISE

ON

THE STRENGTH OF BRIDGES AND ROOFS.

CHAPTER I.

1. **Truss.**—The term Truss is generally applied in Engineering to a frame work constructed to transfer its own weight and a weight imposed upon it to the supports or abutments on which it rests, and whose members are subject to longitudinal strains only.

2. **Strains.**—The strains affecting a truss are of but two kinds; compression or thrusting, and tension or pulling.

3. **Chords.**—A chord is the outer longitudinal continuous member of a truss. There are two chords in a truss; an upper and a lower chord.

4. **Braces.**—Braces are the members of a truss connecting the chords.

5. **Ties.**—Ties are those braces which are subject to tension.

6. **Struts.**—Struts are those braces which are subject to compression.

7. **Panels.**—The term panel is now generally applied

to the divisions of the truss which are formed by the vertical braces, or by the intersections of the braces with the chords.

The investigation of strains in trusses may be based upon the three following laws, given and demonstrated in elementary works on Mechanics:

8. The Lever.—*If a weight be borne by a beam or truss, resting at its extremities upon two supports, these supports may be considered as reacting with two upward pressures, whose sum is equal to the weight; and the weight borne by either support, or the reaction of either support, is to the whole weight as the distance from the centre of gravity of the weight to the farther support, is to the whole length of the beam or truss.*

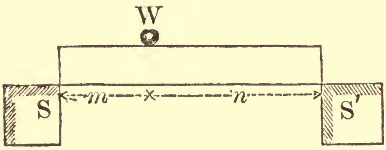


Fig. 1.

Thus, let W, Fig. 1, represent the weight borne by the beam or truss, S and S′ the reactions of the supports, m and n the distances of the weight from the supports, and l the length of the beam or truss which equals $m + n$; then, by this law,

$$S + S' = W$$

$$S' : W :: m : l$$

$$\therefore S' = \frac{Wm}{l} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, *the reaction of one abutment or support is equal to the whole weight, divided by the length of the beam, and mul-*

tiplied by the distance of its centre of gravity from the other support.

This principle of the lever cannot be affected by any shape of the beam or by any bracing within a truss.

Ex.—Let W = 12 tons, $m = 6$ feet, and $n = 3$ feet;

$$\therefore S' = \frac{12 \times 6}{6+3} = 8 \text{ tons},$$

and

$$S = \frac{12 \times 3}{6+3} = 4 \text{ tons}.$$

9. Resolution of Forces.—*If three forces acting at one point balance; three lines parallel to their directions will form a triangle whose sides will be proportional to the forces.*

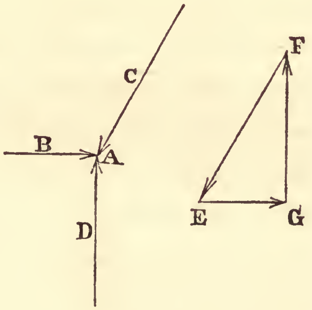


Fig. 2.

Let the lines B, C and D, represent three forces, either pulling or thrusting, and balancing each other at A. Draw EF parallel to C to any scale, and through E and F, draw EG and FG parallel to B and D. Then the length of the sides EF, FG, and EG are proportional to the amounts of the forces, C, B and D. Again, if C be a force acting at A whose amount and direction are represented by the line EF; EG will represent its horizontal component, or force in a horizontal direction, and FG its vertical component or force in a vertical direction; or a force may be resolved into two components, acting in the lines of and equal to the two forces which keep it in equilibrium.

Hence, it is evident that if we know either the vertical or horizontal component of an inclined force and its inclination, we may determine its amount and its other component; or if the force and its inclination be known, its horizontal and vertical components may be readily found; for in either case we have the angles and one side of a right-angled triangle.

If three members of a truss or frame meet as at A, Fig. 2, the strains to which they all are subject are of the same character, either all compression or all tension. If two of the three members of a truss meeting at one point are on the same side of the line of the third, they are subject to different strains: the outer members, or those which make the greater angle with each other, having the same kind of strain, and the interior member the opposite strain.

Thus, in Fig. 3, if B be subject to tension, C is likewise

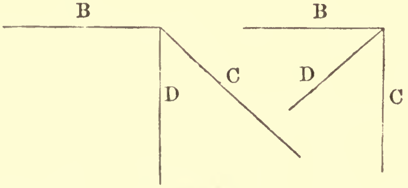


Fig. 3.

subject to tension and D to compression, and if D be subject to tension, B and C are subject to compression.

10. The Equality of Moments.—*The moments of the forces or strains acting upon a body in equilibrium which tend to turn it in one direction about a certain point are equal to the moments of the forces or strains which tend to turn it in the opposite direction; the forces and the point being in the same plane.*

The moment of a force at any point is its amount multiplied by its distance at right angles to its direction from the point about which the moments are taken. A force whose line of direction passes through a point about which moments are taken has no moment at that point.

In Fig. (1), taking moments about the left support, we have W, the force acting downward multiplied by m, its perpendicular distance, for the moment of the weight in one direction; and S′, the reaction of the right support acting upwards, multiplied by l, its perpendicular distance, for the moment of the force in the opposite direction: S having no moment. Hence this equation may be formed,

$$Wm = S'l,$$

and

$$S' = \frac{Wm}{l},$$

as before.

11.—In trusses, each member may represent the line of some force or strain, and to take moments with accuracy, it is necessary to select a point in a vertical section cutting only one member of the truss whose line of direction does not pass through the point.

Thus, in Fig. 4, the moments may be taken around *b* in the vertical section *db*, to determine the strain in *cd*; because the strains in *db* and *ab*, passing through

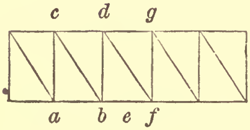


Fig. 4.

b, have no moment, and consequently do not enter the equation. But if the moments be taken around *e* between *b* and *f*, the vertical section will cut *df* and *dg*, and the two unknown strains contained in them will enter the equation and render it indeterminable.

CHAPTER II.

CASE I.—A SIMPLE TRUSS SUPPORTED AT THE ENDS, AND LOADED AT THE CENTRE ONLY.

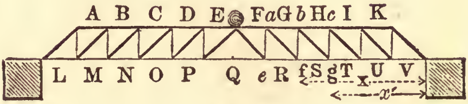


Fig. 5.

12.—Let w = the weight upon the centre,

l = the length of the truss,

d = the depth of the truss from centre to centre of the chords,

x and x' = horizontal distances from one abutment to the vertical braces or ends of a panel,

H and H′ = horizontal strains in either chord at the points x and x',

V = vertical strains, which affect the braces only.

The strains caused by any load in the upper chord of a truss of a single span can evidently be compression only, while those in the lower chord can be tension only.

13. The Horizontal Strain.—In this case the weight borne by either abutment or the reaction of either abutment is evidently $\frac{w}{2}$. The segment to the right of a vertical section through bf, for example, is kept in equilibrium by the reaction of the right abutment and the strains at b and f.

Taking moments around f distant x from the right abutment, we have the reaction of the abutment, $\frac{w}{2}$, multiplied by x, or $\frac{wx}{2}$, for the moment in one direction; and in the opposite direction we have only the strain H at b, which, multiplied by its distance, d, gives the moment Hd.

Whence $$Hd = \frac{wx}{2},$$

And, $$H = \frac{wx}{2d}, \quad . \quad . \quad . \quad . \quad (2)$$

14.—In Eq. (2) H varies directly as x, and is greatest, when x is greatest, that is, when it is equal to $\frac{l}{2}$; at the abutment becomes zero, and varies at any point directly as the weight and inversely as the depth.

15.—The amount of horizontal compression at the point b, shown by Eq. (2), is, on the left, the strain in ab; and on the right the strain in bc and the horizontal component of the strain in bg; and that a strain exists in the latter may be shown as follows: Take moments similarly around g, distant x' from the right abutment and we obtain,

$$H' = \frac{wx'}{2d}, \quad . \quad . \quad . \quad . \quad (3)$$

for the compression in bc; less than the compression in ab, because x' is less than x.

Subtracting Eq. (3) from Eq. (2) we have,

$$H - H' = \frac{w}{2d}(x - x'), \quad . \quad . \quad . \quad (4)$$

for the excess of compression in *ab* over that in *bc*. This force may be considered as the sole horizontal force acting at *b*, for the remainder of the strain or the thrust at that point in *ab* balances that in *bc*. We have therefore a thrust, H — H′, towards *b*, which must be balanced by the strains in *bf*, and *bg*, as they are the only members meeting at *b*. Therefore, the strain in *bg* is (9) compression, and the strain in *bf* is tension; and the horizontal component of the former is equal to H — H′, the horizontal force at *b*, and its vertical component is equal to the strain in *b f*.

16. The Vertical Strain.—Let the length of the strut *bg* represent the longitudinal strain to which it is subject; *bc* will therefore represent its horizontal component, or the value of H — H′, and *bf* its vertical component or the amount of tension in that tie; the latter may then be obtained from the former by the following proportion:—

$$bc : bf, \text{ or } x - x' : d :: \frac{w}{2d}(x - x') : \frac{w}{2}.$$

Whence $$V = \frac{w}{2}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

is the vertical strain in *bf* and the vertical component of the strain in *bg*; and, since it is a constant, the vertical strain in all the braces. It is likewise independent of the length and depth of the truss, and is equal to the reaction of the abutment.

17.—The horizontal component of the strain in the struts, $\frac{w}{2d}(x - x')$, is a constant, if the length of the panels, $x - x'$ is uniform; but unlike the vertical strain is affected by the depth of the truss and by the inclination of the braces, or the length of the panels.

18.—The longitudinal strain in the struts is readily determined from the horizontal or vertical component: from the latter as follows:—

$$d : bg :: \frac{w}{2} : \frac{w\,(bg)}{2d},$$

a constant if the struts be of uniform length.

19.—The tension in any member of the lower chord is determined similarly to the compression in the upper chord. Taking moments around b, we have, as before,

$$\mathrm{H} = \frac{wx}{2d},$$

for the tension at f, the whole of which is contained in fg, and at g the tension is,

$$\mathrm{H}' = \frac{wx'}{2d}, \text{ contained in } gh.$$

Their difference is an excess of tension in fg which gives us as before, tension in cg and compression in bg; the amounts of which can be determined from the changes in the amounts of strain in the lower chord as well as from those in the upper chord.

Hence, the general form of Eq. (2) gives, in the case supposed, the horizontal strains in the upper chord on the side towards the centre of the points to which x is measured, and in the lower chord on the abutment side of the same points, and x may be measured from either end to the weight. It will be noticed that when the upper ends of the braces are inclined towards the weight they are struts, when vertical, ties.

20.—Example: Let Fig. 5 represent a truss sixty feet long and five feet deep, divided into twelve panels of uniform length and supporting a weight of eighty tons at the centre.

Then, $w = 80$ tons,

$l = 60$ feet,

$d = 5$ feet.

$x - x' = 5$ feet, a panel length.

$$\text{Length of struts} = \sqrt{5^2 + 5^2} = 7.07.$$

Whence $V = \frac{w}{2} = 40$ tons, tension in all the ties.

$\frac{w\,(bg)}{2d} = \frac{80 \times 7.07}{2 \times 5} = 56.56$ Tons, compression in all the struts.

$$H = \frac{wx}{2d} = \frac{80\,x}{2 \times 5} = 8\,x.$$

The different values of x or distances to the ends of the panels are, 5, 10, 15, 20, 25, 30.

In the following table the first line gives the values of x, the second line the amount of strain in tons, and the third and fourth lines the chord members subjected to the strains in the same column:

Values of x.	5	10	15	20	25	30
Strains in Tons.	40	80	120	160	200	240
Compression in.	A & K	B & I	C & H	D & G	E & F	
Tension in.	L & V	M & U	N & T	O & S	P & R	Q

There is no single member at the centre which takes all the strain in the upper chord.

CASE II.—A SIMPLE TRUSS SUPPORTED AT THE ENDS AND LOADED AT A POINT BETWEEN THE CENTRE AND ONE ABUTMENT.

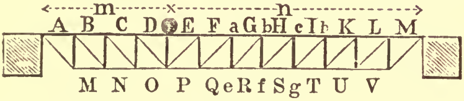


Fig. 6.

21.—Let $w =$ the weight upon the truss,

$l =$ the length of the truss,

$d =$ the depth of the truss,

$x =$ the distance of one of the abutments to any one of the vertical braces or end of a panel,

$p =$ the horizontal length of a panel,

$m =$ the distance of the weight from the left abutment,

$n =$ the distance of the weight from the right abutment,

H and $H' =$ the horizontal strains in the chords,

$V =$ the vertical strain.

By the principles of the lever (8), the reaction of the right abutment is $\frac{wm}{l}$. The segment to the right of a vertical section through any panel end b, is held in equilibrium by the reaction of the right abutment and the strains at b and f. Taking moments around f, distant x

from the right abutment, we obtain, by the same reasoning as in the previous case,

$$H = \frac{wmx}{dl}, \quad . \quad . \quad . \quad . \quad (6)$$

for the horizontal strain at b, or the amount of compression in bc. The value of H in this equation varies directly as x, and is greatest when x is greatest or equal to n, that is under the weight. If the moments to the left of w and the reaction of the left abutment be taken we shall have,

$$H = \frac{wnx}{dl}, \quad . \quad . \quad . \quad . \quad (7)$$

x being measured from the left abutment. These equations also give the tension in the lower chord. If another point be taken in the section through cg, one panel length nearer the abutment, we shall have,

$$H' = \frac{wm\,(x-p)}{dl}. \quad . \quad . \quad . \quad (8)$$

A value evidently less than the value of H in Eq. (6), showing that there is in bc an excess of compression over the amount in ch. This excess may, as in the previous case (15), be considered as the sole horizontal force at c, and is equal to the horizontal component of the strain in cf, which strain is (9) tension. The strain in cg is consequently compression and is equal to the vertical component of the strain in cf.

Subtracting Eq. (8), from Eq. (6), we have,

$$H - H' = \frac{wmp}{dl} \quad . \quad . \quad . \quad (9)$$

for the horizontal component of the strain in the tie cf, and as it is a constant and independent of any value of x, for the horizontal component of the strain in any tie between the weight and the right abutment. Similarly, $\frac{wnp}{dl}$ is the horizontal component of the strain in the ties between the weight and the left abutment.

22. Vertical Strain.—The vertical component may be obtained from the horizontal component of the strain in the ties to the right of the weight, by the proportion used before:

$$p : d :: \frac{wmp}{dl} : \frac{wm}{l}.$$

Whence, $$V = \frac{wm}{l}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

is the vertical component of the strain in each of the ties, and the total compression in each of the struts in the right segment;

And similarly, $$V = \frac{wn}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

is the vertical component of the strain in each of the ties and the total compression in each of the struts in the left segment.

23.—Hence we obtain this law, equally applicable to the previous case: *The vertical strain in either segment of a truss loaded at one point only is equal to the reaction of the abutment on which the segment rests.*

24. Longitudinal Strain.—The longitudinal strain in the inclined braces may be, as before, obtained from this proportion; as the depth of the truss is to the length of the brace, so is the vertical reaction of the abutment to the strain; or,

$$d : be :: \frac{wm}{l} : \frac{wm\,(be)}{dl};$$

the tension in the ties to the right of the weight. And similarly the tension in the ties to the left of the weight may be found.

25.—EXAMPLE: Let Fig. 6 represent a truss sixty feet long and four feet deep, divided into twelve panels of uniform length and supporting a load of sixty tons at the distance of twenty feet from the left abutment.

$$\begin{aligned} \text{Here} - w &= 60 \text{ Tons}, \\ l &= 60 \text{ Feet}, \\ d &= 4 \quad \text{"} \\ m &= 20 \quad \text{"} \\ n &= 40 \quad \text{"} \end{aligned}$$

$$\mathrm{H} = \frac{wmx}{dl} = \frac{60 \times 20 \times x}{4 \times 60} = 5\,x.$$

Substituting the different values of x or the distances from the right abutment to the panel ends, we obtain the following strains in the right segment. This table is arranged as was the previous one.

Values of x.	5	10	15	20	25	30	35	40
Strains in Tons.	25	50	75	100	125	150	175	200
Compression in.	M	L	K	I	H	G	F	E
Tension in.	W	V	U	T	S	R	Q	

For the left segment,

$$\mathrm{H} = \frac{wnx}{dl} = \frac{60 \times 40 \times x}{4 \times 60} = 10\,x,$$

substituting the different values of x, or the distances from the left abutments to the panel ends, we have the following:—

Value of x.	5	10	15	20
Strains in Tons.	50	100	150	200
Compression in.	A	B	C	D
Tension in.	N	O	P	

The length of the ties is $\sqrt{4^2 + 5^2} = 6.4$ feet,

$$\therefore \frac{wm(be)}{dl} = \frac{128}{4} \text{ tons,} = 32 \text{ tons}$$

The tension in each of the ties between the weight and the right abutment.

And $$\frac{wn(6.4)}{dl} = \frac{256}{4} \text{ tons.} = 64 \text{ tons.}$$

The tension in each of the ties between the weight and the left abutment.

$$\frac{wm}{l} = 20 \text{ tons,}$$

Compression in each of the struts to the right of the weight,

$$\frac{wn}{l} = 40 \text{ tons,}$$

Compression in each of the struts to the left of the weight. The strut immediately under the weight is subject to both amounts or 60 tons compression.

26.—If x in Eq. (6) be made equal to n or in Eq. (7) equal to m, we have in either case,

$$H = \frac{wmn}{dl} \quad . \quad . \quad . \quad . \quad (12)$$

for the horizontal strain under the weight; and if x in Eq. (2) be made equal to $\frac{l}{2}$ we have

$$H = \frac{wl}{4d} \quad . \quad . \quad . \quad . \quad (13)$$

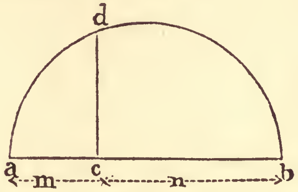


Fig. 7.

In Fig. 7. upon ab, made equal to l, describe a semicircle with a radius equal to $\frac{l}{2}$; then the vertical cd will be a mean proportional between m and n, or

$$m : cd :: cd : n$$

Whence $$mn = (cd)^2$$

Consequently, *in a truss, loaded with a single weight, if the weight be placed at different points, the horizontal strains resulting therefrom under the load, are to each other as the squares of the vertical distances at the points where the weight is placed, between a horizontal line equal in length to the truss and a semicircle inscribed thereon with a radius equal to half the length of the truss.*

27.—The horizontal strain in either of the above cases between the weight and the abutment decreases at the panel ends in uniform quantities, and at these points passes through the inclined braces from one chord to the other where it neutralizes an equal amount of the opposite strain; or *the braces contain all the vertical strain* *and an amount of horizontal strain equal to the total strain in one chord.*

In the first example above, the compression passes down the struts to the lower chord; in the second, the tension passes up the ties to the upper chord; hence, *in a truss there is no neutral axis or line in which no horizontal force exists.*

CASE III.—A TRUSS UNIFORMLY LOADED THROUGHOUT ITS LENGTH.

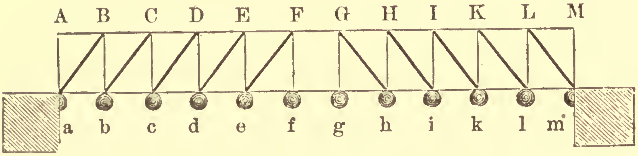


Fig. 8.

28.—Let w = the whole weight upon the truss.

l = the length of the truss,

d = the depth of the truss,

x = the distance to the end of a panel from one abutment.

p = the length of a panel.

u = the distance from the same abutment from which x is measured, equal to $x - \frac{p}{2}$, being the distance to the centre of a panel.

H = the horizontal strain.

V = the vertical strain.

In a truss the load is to be considered as concentrated at the ends of the panels, for it is at these points that the connections are made between the truss and the members of the bridge which receive the load. A half panel load is therefore borne by either abutment.

29. Horizontal Strain.—The weight upon either abutment is $\frac{w}{2}$. The segment to the right of any vertical section, Hh, is held in equilibrium by the reaction of the right abutment, the load on hm, and the strains at H and h. Taking moments around h distant x from the right abutment, we have the moment of the right abutment, equal to $\frac{wx}{2}$, in one direction, and the load on the segment to the right of Hh multiplied by the distance of its centre of gravity from h, for the moment in the opposite direction; their difference is the horizontal strain at H multiplied by its distance d.

The moment of the load on x may be found as follows: The weight coming directly upon the abutment is half a panel load, $\frac{wp}{2l}$, its moment is therefore $\frac{wpx}{2l}$; the weight upon h has no moment, and the weight upon the remainder of the segment is evidently $\frac{w}{l}$ multiplied by $x - p$, and the distance of its centre of gravity is $\frac{x}{2}$; whence its moment is $\frac{w}{l}(x-p)\frac{x}{2} = \frac{wx^2}{2l} - \frac{wpx}{2l}$, or the moment of the whole load on x is $\frac{wx^2}{2l} - \frac{wpx}{2l} + \frac{wpx}{2l} = \frac{wx^2}{2l}$; we can therefore form, for the horizontal strain, the following equation:—

$$Hd = \frac{wx}{2}, - \frac{wx^2}{2l},$$

Whence
$$H = \frac{wx}{2d} - \frac{wx^2}{2dl}, \quad . \quad . \quad . \quad (14)$$

(The half panel load resting directly upon the abutment has been generally disregarded in calculating the strains in trusses as it does not affect the results; but it has here been introduced because the equations are rendered simpler, and w represents thereby the whole load upon the truss).

Eq. (14) gives the compression in members of the upper chord, in the form of truss shown on the centre side of the panel end to which x is measured and the tension in the lower chord members on the abutment side of the same point, and is not confined as in the last preceding case to points between the centre of gravity of the load and the abutment, but is true for any value of x, which may be measured from either end.

Differentiating Eq. (14), we shall find that H attains its maximum value when $x = \frac{l}{2}$, or at the centre; where, substituting $\frac{l}{2}$ for x, we have,

$$H = \frac{wl}{8d}, \quad . \quad . \quad . \quad . \quad (15)$$

Comparing this with Eq. (13) we see, *that the horizontal strain at the centre of a uniformly loaded truss is one half what it would be if the same load were concentrated at the centre.*

At the abutment H becomes zero.

30.—EXAMPLE: Let Fig. 8 represent a truss 110 feet long, 12.5 feet deep, divided into eleven panels of uniform length, loaded on the lower chord with a load of fifteen tons per running foot, or a total load of 165 tons.

$$\text{Here} \qquad l = 110 \text{ feet},$$
$$d = 12.5 \text{ ''}$$
$$w = 165 \text{ tons}.$$

Substituting the values of these constants in Eq. (14), we have,

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl} = \frac{165\,x}{2 \times 12.5} - \frac{165\,x^2}{2 \times 12.5 \times 110} = 6.6x - .06\,x^2$$

Whence the horizontal strains in the chords are as follows:—

Values of x	10 or 100	20 or 90	30 or 80	40 or 70	50 or 60
Strains in Tons.	60	108	144	168	180
Compression in.	KL & BC	IK & CD	HI & DE	GH & EF	FG
Tension in.	*lm & ab*	*kl & bc*	*ik & cd*	*hi & de*	*ef, fg & gh*

31.—Eq. 14, is the equation of a segment of a parabola whose diameter is equal to $\frac{l}{2}$ referred to rectangular axes, whose origin is at the intersection of the diameter with the curve.

Let AX and AY, Fig 9, represent the axes, H being measured on AX and x on AY. Make $x = 0$, and H become zero, or the curve passes through the origin A; make $x = l$ its maximum value and again H = 0, or the curve intersects AY at the distance l from A; make $x = \frac{l}{2}$ and $H = \frac{wl}{8d}$, its maximum value at the

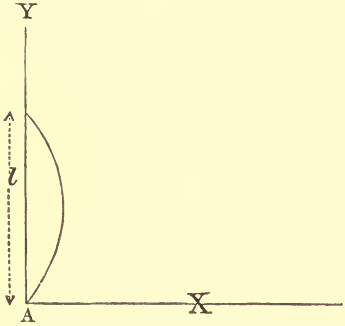


Fig. 9.

vertex of the curve; and at any point the distance between AY and the curve represents the strain in the truss at that distance from the abutment; practically this may be done as follows:—

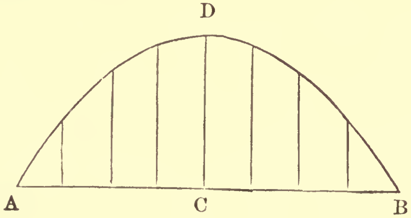


Fig. 10.

Let AB, Fig. 10, be equal to l, the length of the truss, and upon its centre, C, erect a perpendicular whose height is equal to $\frac{wl}{8d}$, by any scale. Then through ADB construct a parabola, and the length of any vertical between AB and ADB by the same scale will give the horizontal strain in either chord at the distance from either abutment that the vertical is from A or B.

32. The Vertical Strain.—Taking Eq. (14),

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl},$$

at the end of any panel distant x from the abutment, the horizontal strain at the next panel end towards the abutment is, since the distance is $x - p$,

$$H' = \frac{w(x-p)}{2d} - \frac{w(x-p)^2}{2dl}. \quad . \quad . \quad (16)$$

Subtracting Eq. (16) from Eq. (14) we have,

$$H - H' = \frac{wx}{2d} - \frac{w(x-p)}{2d} - \frac{wx^2}{2d} + \frac{w(x-p)^2}{2dl},$$

$$= \frac{wp}{2d} - \frac{wp}{dl}\left(x - \frac{p}{2}\right),$$

and since $$x - \frac{p}{2} = u,$$

$$H - H' = \frac{wp}{2d} - \frac{wpu}{dl}, \quad . \quad . \quad (17)$$

The excess of horizontal strain in the upper chord thrusting towards the abutment beyond that thrusting towards the centre and which (9) consequently causes in a truss of this form compression in the inclined and tension in the vertical braces. It is therefore at any point equal to the horizontal component of the strain in the strut at the same point, whose vertical component is equal to the tension in the tie which meets it at the upper chord. This vertical strain can, as before, be obtained from the proportion,

$$p : d :: \frac{wp}{2d} - \frac{wpw}{dl} : \frac{w}{2} - \frac{wu}{l}$$

$$\therefore \quad V = \frac{w}{2} - \frac{wu}{l}. \quad . \quad . \quad . \quad (18)$$

Whence—*The vertical strain at any point in a uniformly loaded truss is equal to the weight borne by one abutment, less the weight between that point and the abutment:* the point being measured from the nearest abutment. Eq. (18) varies inversely with the different values of u, being zero when $u = \frac{l}{2}$ or at the centre, and $\frac{w}{2}$, when $u = 0$, or at the abutment.

Substituting the values of the constants in the above example, Eq. (18) becomes,

$$V = 82.5 - 15.\ u,$$

and the different values of u are, 5, 15, 25, 35, 45, whence we obtain the following table:—

Values of u.	5	15	25	35	45
Strains in Tons.	75	60	45	30	15
Tension in.	Bb & Ll	Cc & Kk	Dd & Ii	Ee & Hh	Ff & Gg

When $u = 55$, $V = 0$, or there is no strain at the centre.

33. Longitudinal Strains in the Struts.—The longitudinal strain in any strut is the vertical strain divided by the depth of the truss and multiplied by the length of the strut. Performing this operation with Eq. (18) we have, since the length of a strut is 16 feet,

$$L = 105.6 - 1.92\ u. \quad . \quad . \quad . \quad . \quad . \quad (19)$$

whence we can form the following table of compression in the struts:

Values of u.	5	15	25	35	45
Strains in Tons.	96	76.8	57.6	38.4	19.2
Compression in.	Lm & Ba	Kl & Cb	Ik & Dc	Hi & Ed	Gh & Fe

34.—In Fig. 8, at the centre, there is, as is shown by the Eq. (18), no vertical strain; then taking the weight on the next panel point g, 15 tons, we find it produces a tension equal to its amount in Gg, and a vertical strain, or vertical component of a strain, of the same amount in Gh; there the weight on the next panel point, h, is added to it, and the vertical strain in Hh and also in Hi, is equal to the sum of the two weights, and so on to the end of the truss.

Now, suppose the load to be placed on the upper chord; then the weight would be at G, and there would be no strain in Gg; but the vertical strain, or the vertical component of the strain in Gh, would be the same as the weight on G, and would equal the tension in Hh.

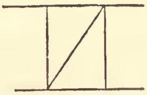


Fig. 11.

If the braces were inclined the opposite way, as in Fig. 11, representing a panel to the right of the centre, and the load upon the lower chord, there would be no strain upon *ac*, but the vertical strain in *cb* would equal that in *bd*; if the load was upon the upper chord, the vertical strain in ac would equal that in bc; or, in any case, *the vertical strain is constant in the braces between the weights.* The vertical strain, or the vertical component of the strain, in the inclined braces remains the same, whether the truss be loaded upon the upper or upon the lower chord, and whatever may be the inclination of the brace.

35.—Therefore, Eq. (18) *gives the vertical strain, or the vertical component of the strain, in the inclined brace, whose centre is distant u from the abutment, and the vertical or total strain in the vertical brace attached to the unloaded end of the inclined brace.*

36.—At the centre, as stated before, there is no vertical strain; or, the vertical strain on either side of the centre of a uniformly loaded truss, passes to the abutment on that side, and if u in Eq. (18) be made greater than $\frac{l}{2}$, V will have a minus value, whence this rule: *In a vertical equation, when V has a plus value, the vertical strain given thereby is passing to the abutment from which u is measured, when a minus value, to the opposite abutment.*

37.—Eq. (18.)

$$V = \frac{w}{2} - \frac{wu}{l},$$

is the equation of a straight line referred to rectangular axes. Let AX, Fig. 12, on which u is measured, and AY on which the values of V are measured, represent the axes; when $u = 0$, then $V = \frac{w}{2}$; and make

AB to any scale equal to $\frac{w}{2}$, when $u = \frac{l}{2}$, $V = 0$, therefore, lay off AC on AX equal to $\frac{l}{2}$, AD equal to l. Draw DE parallel and equal to AB and join BE. On AD, at the distances from A represented by the values

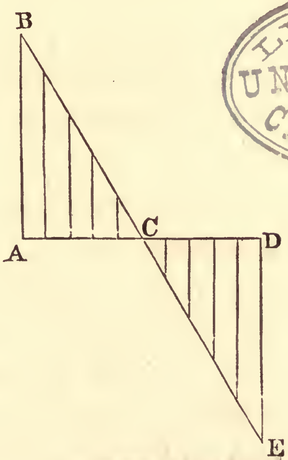


Fig. 12.

of u, erect perpendiculars meeting the line BE, the lengths of these lines will give, by the same scale by which AB was measured, the values of V at the different points. The lines above AB showing the strains passing to the left abutment, those below, the strains passing to the right abutment.

38.—The horizontal and vertical equations given above show, that *where the horizontal strain is greatest there is no vertical strain, and where the vertical strain is greatest the horizontal strain is least.*

CASE IV.—A TRUSS LOADED FROM ONE ABUTMENT ONLY A PORTION OF THE LENGTH.

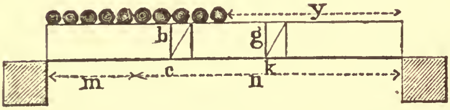


Fig. 13.

39.—Let w = the whole weight upon the truss uniformly distributed, extending from one abutment a distance equal to $2\ m.$,

l = the length of the truss,

d = the depth " "

p = the length of a panel,

m = the distance of the centre of gravity of the load from the loaded abutment,

n = the distance of the centre of gravity of the load from the unloaded abutment,

y = the length of the unloaded part,

x = the distance of any panel end from the unloaded abutment,

H = the horizontal strain,

V = the vertical strain,

L = the longitudinal inclined brace strain.

40. Horizontal Strain in the Unloaded Part.—By the principles of the lever, (8), $\frac{wm}{l}$ is the reaction of the right abutment, and the segment to the right of any vertical section at a panel end, gk, in the unloaded part, is held in equilibrium by the reaction of the abutment, and the strains at g and k. Taking moments around k distant x from the unloaded abutment, we have,

$$\mathrm{H} = \frac{wmx}{dl}, \qquad (20)$$

for the strain at g, or similar to the strain in the case of the truss loaded at one point only, (21). In the lower chord the tension at k is evidently the same.

41. Vertical Strain in the Unloaded Part.—The vertical strain is also plainly $\frac{wm}{l}$ or the reaction of the unloaded abutment, as in the similar case, (21).

42.—In the above operations w expresses the weight of the load upon the truss. It is more convenient and sometimes necessary that the equation should express the weight of a full load of uniform density with the partial load.

Let $w' =$ the weight of a full uniform load of equal density with the partial load. Then $w = \frac{w'}{l}(l—y)$ and $m = \frac{l—y}{2}$, and $\therefore \frac{wm}{l} = \frac{w'(l—y)^2}{2l^2}$.

43. Horizontal Strain in the Loaded Part.—The segment to the right of any panel end, bc, in the loaded part is held in equilibrium by the reaction of the right abutment, the load on $x—y$, which is $\frac{w'}{l}(x—y)$, and the strains at b and c. Taking moments around c distant

x from the right abutment, we have therefore, $\frac{x-y}{2}$ being the distance from c of the centre of gravity of the load on $x-y$,

$$Hd = \frac{w'(l-y)^2 x}{2l^2} - \frac{w'}{l}(x-y)\frac{x-y}{2}.$$

Whence

$$H = \frac{w'(l-y)^2 x}{2dl^2} - \frac{w'(x-y)^2}{2dl}, \quad - \quad - \quad (21)$$

is the compression in the upper chord at b.

The same result may be obtained for the tension in the lower chord, at c.

At a point one panel length nearer the right abutment,

$$H' = \frac{w'(l-y)^2(x-p)}{2dl^2} - \frac{w'(x-p-y)^2}{2dl}. \quad - \quad - \quad (22)$$

44. Vertical Strain in the Loaded Part.—The vertical strain may be obtained, as in the previous cases, from the difference in the horizontal strains at the different ends of the same panel. Subtracting Eq. (22) from Eq. (21) we have,

$$H-H' = \frac{w'p}{2dl^2} - \frac{w'py}{dl} + \frac{w'px}{dl} - \frac{w'p^2}{2dl},$$

And $\quad p : d :: H-H' : \frac{w'(l-y)^2}{2l^2} - \frac{w'}{l}(x - \frac{p}{2} - y).$

And since $x - \frac{p}{2} = u$

$$V = \frac{w'(l-y)^2}{2l^2} - \frac{w'}{l}(u-y). \quad - \quad - \quad - \quad (23)$$

In this equation, the less $u-y$ becomes, the greater is the value of V, and the latter is greatest when $u-y$ becomes zero, or in the brace at the end of the load, where it equals $\frac{w'(l-y)^2}{2l^2}$ or the reaction of the abutment. Again, V decreases with any increase of $u-y$, and there may be a point where $V = 0$; to find this point, make V of Eq. (23) equal to zero,

$$\therefore \frac{w'(l-y)^2}{2l^2} = \frac{w'}{l}(u-y)$$

and $u = \frac{l^2+y^2}{2l.}$ is the distance from the unloaded abutment to the point where there is no vertical strain. To the right of this point V has a positive value and the vertical strain passes to the right abutment; to the left, a negative value, and the vertical strain passes to the other abutment. If Eq. (21) be differentiated to find the maximum value of H, it will be found to be when $x = \frac{l^2+y^2}{2l}$ or in the same panel, in which there is no vertical strain. Another proof of the rule in (38).

45.—In Eq. (23) u cannot equal $\frac{l^2+y^2}{2l}$ until $\frac{w'(l-y)^2}{2l^2} = \frac{w'}{l}(u-y)$, or until we have passed an amount of the load equal to that borne by the abutment; hence this important rule: *There is a point in every fully or partially loaded truss where there is no vertical strain, but where the horizontal strain is greatest, which divides the load into the two parts borne by the two abutments, the part on either side of this point being borne by the abutment on that side.*

Therefore, knowing the load borne by either abutment, we have only to pass from the abutment along the load, full or partial, until we measure an equal weight, and we reach the point of no vertical strain.

Example.—If a truss be 50 feet long, and is loaded from one abutment a distance of 30 feet, at the rate of 1.5 ton per foot, where is the point of no vertical strain?

Here, $w' = 75$ tons,

$l = 50$ feet,

$y = 20$ feet,

$$\therefore \frac{w'(l-y)^2}{2l^2} = \frac{75\ (50-20)^2}{2 \times 50^2} = 13.5 \text{ tons},$$

the reaction of the unloaded abutment.

Beginning at the end of the load towards the unloaded abutment, which is 20 feet from that abutment, we must go from the end of the load 9 feet towards the other end of the truss before we have passed 13.5 tons and reached the point of no vertical strain. And by the formula found above

$$\frac{l^2+y^2}{2l} = \frac{2500+400}{100} = 29 \text{ feet from the abutment.—}Ans.$$

No vertical strain can pass this point which can exist at one place only in any truss, no matter how loaded. Each abutment's share of the weight comes directly from *that part of the load nearest to it; and vertical strains in the same truss cannot pass each other.*

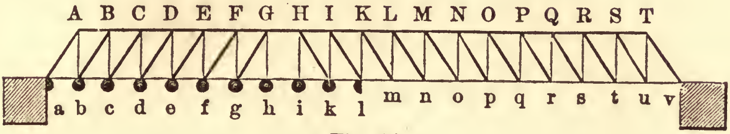


Fig. 14.

46.—Let Fig. 14 represent a truss 80 feet long, 6 feet deep, divided into 20 uniform panels, and supporting on the lower chord a load of 25 tons, extending from the left abutment to the centre of the truss.

Here, $w = 25$ tons, $\therefore\ w' = 50$ tons,

$l = 80$ feet,

$d = 6$ feet,

$p = 4$ feet,

$y = 40$ feet,

Length of inclined braces = 7.2 feet.

The equation for the horizontal strains in the unloaded part is,

$$H = \frac{w'x(l-y)^2}{2dl^2} = \frac{50x(80-41)^2}{2\times6\times80^2} = 1.0417x.$$

Whence the following table of the strains in the chords:

Values of x	4	8	12	16	20	24	28	32	36	40
Strains in Tons.	4.2	8.3	12.5	16.6	20.8	25	29.2	33.3	37.5	41.7
Compression in.	ST	RS	QR	PQ	OP	NO	MN	LM	KL	IK
Tension in.	uv	tu	st	rs	qr	pq	op	no	mn	lm

The equation for the horizontal strains in the loaded part is

$$H = \frac{w'x(l-y)^2}{2dl^2} - \frac{w'(x-y)^2}{2dl} = \frac{50x(80-40)^2}{2\times60\times80} - \frac{50(x-40)^2}{2\times60\times80} = 1.0417x - .0521(x-40)^2.$$

Whence the following table:

Values of x.	44	48	52	56	60	64	68	72	76
Strains in Tons.	45	46.7	46.7	45	41.7	36.7	30	21.7	11.7.
Compression in.	HI	GH	GH	FG	EF	DE	CD	BC	AB
Tension in.	kl	ik	hi&gh	fg	ef	de	cd	bc	ab

The equation for the vertical strain in the unloaded part is

$$V = \frac{w'(l-y)^2}{2l^2} = \frac{50\,(80-40)^2}{2\times(80)^2} = 6.25 \text{ tons,}$$

tension in all the vertical ties from Kl to Tu inclusive, and the vertical component of the strain, which is from their inclination, compression, in the inclined braces of the unloaded part.

$L = V \times$ length of strut divided by $d = \frac{7.2V}{6}$ $= 7.5$ tons, compression in the struts from Km to Tu inclusive.

The equation for the vertical strain in the loaded part is,

$$V = \frac{w'(l-y)^2}{2l^2} - \frac{w'(u-y)}{l} = 31.25 - .625u.$$

Whence we form the following table for the tensions in the ties in the loaded part:

Values of u	42	46	50	54	58	62	66	70	74	78
Strains in Tons.	5	2.5	0	—2.5	—5	—7.5	—10	-12.5	—15	--17.5
Tension in	Ik	Hi		Gh	Fg.	Ef	De	Cd	Bc	Ab

The equation applies to the ties at the ends of the inclined braces nearest the centre of the truss, for the reasons given in (35).

From the above table it will be seen that there is no vertical strain in the panel GH *hi*, and that to the left of this panel the strain has the minus sign, showing that the weight is now passing to the abutment opposite that from which u is measured. From the previous table it is also seen that the horizontal strain is greatest in the same panel, hence it is from this point that the braces must incline in opposite directions to the abutments.

Multiplying the vertical strain by the length of strut, and dividing it by the depth of the truss, as before, we have the longitudinal strains in the struts in the loaded part as follows:

Values of u	42	46	50	54	58	62	66	70	74	78
Strains in Tons.	6	3	0	—3	—6	—9	—12	—15	—18	—21
Compression in.	Il	Hk		Gg	Ff	Ee	Dd	Cc	Bb	Aa

CASE V.—A TRUSS SUBJECT TO A UNIFORM CONSTANT LOAD THROUGHOUT ITS LENGTH, AND A UNIFORM MOVABLE LOAD.

47.—Let w = the weight of the full constant load,

w' = the weight of the full load whose weight per lineal foot is the same as that of the partial load,

l = the length of the truss,

d = the depth " "

p = the length of a panel,

x = the distance from one abutment to a panel end,

$u = x - \frac{p}{2}$,

y = the length of the unloaded part,

H = the horizontal strain,

V = the vertical strain.

48.—In the previous cases the weight of the truss itself has been entirely disregarded; but this is the case of a truss the weight of which is considered, subject to the action of a rolling load, and is to a certain extent a combination of the two previous cases. Here it is necessary to obtain the maximum strains only to which each member of the truss is subject, whether from a full or partial load.

49. Horizontal Strains.—In Eq. (14) we have,

$$H = \frac{w'x}{2d} - \frac{w'x^2}{2dl},$$

for the horizontal strain under a full load in either chord.

In Eq. (21),

$$H = \frac{w'x(l-y)^2}{2dl^2} - \frac{w'(x-y)^2}{2dl},$$

for the horizontal strain at the same point from a load of equal density but covering only a part of the truss, $l-y$.

Eq. (21) will reduce to this form,

$$H = \frac{w'x}{2d} - \frac{w'x^2}{2dl} - \frac{w'y^2}{2dl}\left(1 - \frac{x}{l}\right), \quad - \quad - \quad (24)$$

which is less than Eq. (14) by the quantity

$$\frac{w'y^2}{2dl}\left(1 - \frac{x}{l}\right),$$

or the horizontal strain at any point is greatest under a full load, no matter how small y may be, or how large a portion of the truss may be loaded. Hence, where w is the constant truss load, and w' the weight of a full rolling load, the equation for the greatest horizontal strain is

$$H = \frac{(w'+w)x}{2d} - \frac{(w'+w)x^2}{2dl}. \quad - \quad - \quad - \quad - \quad (25)$$

50. Vertical Strain.—In Eq. (23) we have,

$$V = \frac{w'(l-y)^2}{2l^2} - \frac{w'}{l}(u-y),$$

for the vertical strain at u, from a partial load reaching from one abutment a distance equal to $l-y$. Let y and u be measured from the right abutment and confining this equation to the right of the point of no vertical strain, or to its positive values, it is evident that, considering y for the moment constant, V is greater as $u—y$ is less, and is greatest when $u—y$ is least, or when $u = y$ or at the end of the load; where, therefore,

$$V = \frac{w'(l-y)^2}{2l^2}. \quad - \ - \ - \ - \ - \ - \ - \ - \quad (26)$$

Or, the vertical strain from a partial load passing in one direction at any point is greatest when the load extending from the abutment reaches to that point.

Eq. (26) will reduce to this form,

$$V = \frac{w'}{2} - \frac{w'u}{l} + \frac{w'u^2}{2l^2}. \quad - \ - \ - \ - \ - \ - \quad (27)$$

The vertical strain at the same point from a full load of equal density is, Eq. (18),

$$V = \frac{w'}{2} - \frac{w'u}{l}.$$

If u in this equation be less than $\frac{l}{2}$ or when V has a positive value, it follows, *That when a truss is partially but more than half loaded, the load extending from one abutment, the vertical strain at any point at the end of the load is greater by* $\frac{w'u^2}{2l^2}$ *than the vertical strain at the same point from a full load of equal density.*

The greatest vertical strain, therefore, in a truss subject to a rolling load, is the strain at any point from the constant truss weight added to the strain from the rolling load when it reaches that point and covers the greater segment of the truss; hence, adding Eq. (18) and Eq. (26),

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'(l-u)^2}{2l^2}, \quad - \quad - \quad (28)$$

y being here equal to u, is the vertical strain from the rolling load w', and constant load w.

51.—In a truss uniformly loaded there is a point of no vertical strain at the centre, and in a truss partially loaded, where the truss weight is disregarded, there is also a point of no vertical strain, distant $\frac{l^2+y^2}{2l}$ from the unloaded abutment (44), but in no case can there be two points of no vertical strain, for a section of the truss cannot support a portion of the weight without transmitting it entirely to one or partially to either abutment. This point of no vertical strain can be found by making Eq. (28) = 0.

$$\text{Whence} \quad V = \frac{w}{2} - \frac{wu}{l} + \frac{w'(l-u)^2}{2l^2} = 0$$

Let $w' = aw$, eliminate w and we have

$$\frac{1}{2} - \frac{u}{l} + \frac{al^2 - 2alu + au^2}{2l^2} = 0$$

Whence,

$$u = l + \frac{l}{a} - l\sqrt{\frac{1}{a} + \frac{1}{a^2}}. \quad - \quad - \quad (29)$$

Example.—In a truss 200 feet long, whose permanent uniform load is 75 tons, and the weight of its full load of equal density with the partial load, 150 tons; how far from the unloaded abutment is the end of the partial load when it is at the point of no vertical strain, or what is the value of u?

$$\text{Here, } \frac{w'}{w} = a = 2.$$

$$l = 200$$

$$\text{Whence, } l + \frac{l}{a} - l\sqrt{\frac{1}{a} + \frac{1}{a^2}}$$

$$= 200 + \frac{200}{2} - 200\sqrt{\tfrac{1}{2} + \tfrac{1}{4}} = 126.8 \text{ ft.}\text{—}\textit{Ans.}$$

That is, when the partial load covers 73.2 feet of the truss, the end of it is at the point of no vertical strain. The weight borne by the farther abutment is equal to the weight of 126.8 feet of the truss, or 47.5 tons. When the truss is unloaded the weight on the abutment is 37.5 tons. When the load of .75 ton per foot covers 73.2 feet, by the principles of the lever, 10 tons are added to the weight on the farther abutment. While the end of the movable load rests at the point of no vertical strain the weight upon the farther abutment comes solely from the weight of the truss, and none from the load upon it.

When the end of the load approaches the unloaded abutment, passing the point referred to of no vertical strain from the fixed and movable loads, the weight upon this abutment is increased, but as the load still covers less than half the truss, the larger portion of the increased load is borne by the nearer abutment; the point of no vertical strain, therefore, does not remain stationary, but moves after the end of the load, and reaches the centre when the load covers the truss.

52. Counterbracing.—It is therefore evident that on the loaded abutment side of this point there can be no vertical strain passing to the farther abutment; consequently, it is only from this point that it is necessary to arrange braces to carry the vertical strain to the farther abutment. The braces between these points on either side of the centre, and the centre itself, are termed counterbraces, and come into use only under the action of moving loads.

53.—Eq. (18) added to Eq. (26), or

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'(l-y)^2}{2l^2} \quad - \quad - \qquad (30)$$

may be represented as in Fig. 15.

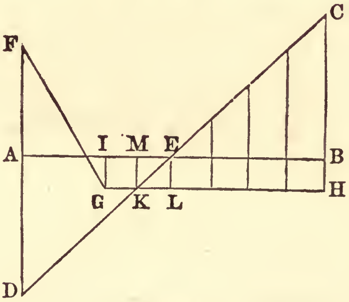


Fig. 15.

Let AB equal the length of the truss; AD and BC each equal to a scale, $\frac{w}{2}$; then (37) the perpendicular distance at any point between EB and EC is the amount of vertical strain from the permanent load at that point passing to the abutment B, and the perpendicular distance between AE and DE at any point represents the amount of vertical strain from the same load, at that point passing to the abutment A; these distances being the values of V in Eq. (18); the positive and negative values being on the opposite sides of AB.

Next, let the load be brought on at A and extend from A to I, and let BH, to the same scale as before, represent $\frac{w'(l-y)^2}{2l^2}$, the reaction of the abutment B, and as this is a constant between I and B, draw I G equal and parallel to BH and join GH. The vertical distance at any point between LH and EC will give the vertical strain from the constant and the movable loads at that point passing to the abutment B and the values of V in Eq. (30) where y is greater and u less than $\frac{l}{2}$.

Let y = BI and let $u = \frac{l}{2}$, then at that point, the centre,

$$V = \frac{w'(l-y)^2}{2l^2}, \text{ or EL.}$$

When u becomes greater than $\frac{l}{2}$, then V is evidently less than $\frac{w'(l-y)^2}{2l^2}$ because $\frac{w}{2} - \frac{wu}{l}$ is a minus quantity to be deducted from it. But in the figure, $\frac{w}{2} - \frac{wu}{l}$ is represented by the vertical distances between AE and DE and $\frac{w'(l-y)^2}{2l^2}$ is represented by the vertical distances between GH and IB; and as $\frac{w}{2} - \frac{wu}{l}$ is a minus quantity to be subtracted from $\frac{w'(l-y)^2}{2l^2}$, then the vertical distance between KE and ME is to be deducted from the vertical distance between KL and ME. Consequently the vertical distance at any point between KC and KH represents the vertical strain at that point passing to the abutment B; and by similar reasoning it may be shown that the vertical strain passing to the abutment A is represented by the vertical distances between DK and the two lines FG and GK. K is the point of no vertical strain.

An examination of Eq. (23) added to Eq. (18), or

$$V = \frac{w'(l-y)^2}{2l^2} - \frac{w'(u-y)}{l} + \frac{w}{2} - \frac{wu}{l},$$

will give the vertical strains under the truss, and is easily made, but has no practical value.

Eq. (26), $V = \frac{w'(l-y)^2}{2l^2}$, is the equation of a parabola, and Eq. (27) may be shown as in Fig. 16.

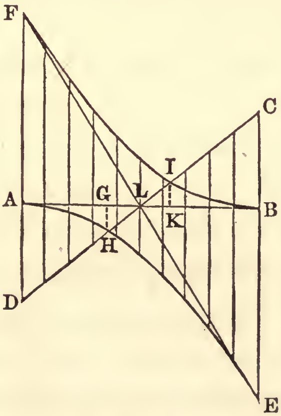


Fig. 16.

Let AB represent the length of the truss, BC and

AD each $\frac{w}{2}$, and AF and BE each $\frac{w'}{2}$; then the vertical distances between DC and FE represent the strains from the weight of the truss and of a full load. Through AE draw a parabola; the vertical distance to any point in the parabola may be found from the value of V in Eq. (26) at that point. Similarly draw the parabola BF. The vertical distances between HC and the curve HE represent the vertical strains from the constant load and the end of the load passing to the abutment B. When the load passes to the left, the vertical strains to A are represented by the vertical distances between ID and the curve IF. Where the curves intersect the line DC, H and I are the points of no vertical strain.

54.—The equation of the moving load, $\frac{w'(l-u)^2}{2l^2}$, requires some change before it can be practically applied to determining strains; because, in its present form, it assumes that no weight comes directly, that is, without passing through the braces, upon the next unloaded panel point. If the load were suspended at each panel point or end, the equation could be applied without change; but as it comes first upon a girder, or string-piece, resting upon the panel points, any load in a panel must affect both ends of the panel.

Let AI (Fig. 17) represent a truss, and A B C D, &c., the different panel ends; and let the load extend from the abutment A to midway between B and C, then there are one and a half panel loads upon the truss; but B does not bear a full panel load, and cannot until the load extends to C, or B cannot have a full load until C has a half load; so that $\frac{w'(l-u)^2}{2l^2}$ is greater than the vertical strain in the brace from B towards the farther abutment, because a part of the strain is in the brace from C.

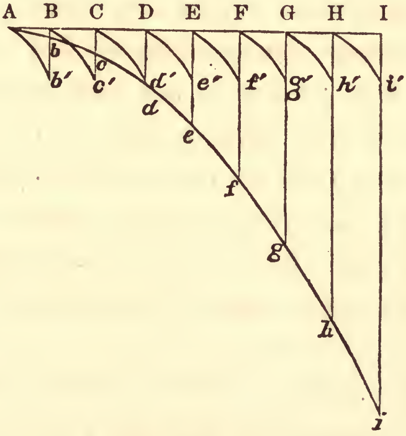


Fig. 17.

Let the vertical distance at any point between AI and the parabola $Abc - - - i$ represent, as shown above, the weight borne by the unloaded, or, in this figure, the right abutment, when the load extends from the left or loaded abutment to that point, and the vertical distances between AI and the smaller similar parabolas Ab' Bc' Cd', &c., represent the weights coming upon the farther panel points B, C, &c., as the load extending from the left abutment traverses the girders resting upon these points. When the load extends from A to B, Bb is the weight borne by I, but Bb' is the weight upon B, which is greater than the weight upon the abutment I; continuing the load from B towards C, the vertical distance from the line BC to the parabola Bc' at any point, will represent the strain upon C, when the load reaches that point, and the vertical distance at the same point between BC and the parabola bc represents the strain at the same time upon the farther abutment, I. When the former equals the latter, or when the curve Bc' intersects the curve bc, all the strain upon I comes from that portion of the load bearing upon C, and none from that portion upon B, and the vertical distance between the two curves, before their intersection, represents the vertical strain upon I that comes from the load upon B; consequently the greatest strain upon the brace at B towards I is when the vertical distance between the two parabolas, between A and B, is greatest. This occurs where a vertical line will intersect the two curves at points where their tangents are parallel; and since the weight on AB is to the whole load upon AI as the distance AB is to the distance AI, the horizontal distances of the tangent points referred to from the panel end and the abutment bear the same proportion to each other.

Let l' represent the length of the partial load, and let it extend from one abutment beyond one panel, to where the tangents to the two curves at points vertically above each other are parallel to each other; let p be the length of a panel, and l the length of the truss; then,

$l : p :: l' : \frac{pl'}{l}$, the distance the load extends into the second panel; and the length of the load l' will therefore be

$$l' = p + \frac{pl'}{l};$$

whence, $$l'l = pl + pl';$$

and $$l' = \frac{pl}{l-p}. \quad \text{- - - - - - -} \quad (31)$$

When the load covers any number, n, of panels, the value of l' will be

$$l' = \frac{npl}{l-p}. \quad \text{- - - - - - - -} \quad (32)$$

Dividing the weight of the whole movable load by the length of the truss, and multiplying by $\frac{npl}{l-p}$, we have the weight of the load on $\frac{npl}{l-p}$, whence (8) the reaction of the unloaded abutment is, in this case,

$$V = \frac{w'}{2l^2}\left(\frac{npl}{l-p}\right)^2. \quad \text{- - - - - - - -} \quad (33)$$

But, as explained before, this is greater than the strain in the braces from the last loaded panel point, because a certain amount of the load is borne by the panel point beyond the load. Hence this last weight is to be deducted from Eq. (33).

The distance from the loaded abutment to the farthest panel end or point which is under the load is np, therefore,

$$\frac{npl}{l-p} - np$$

is the distance the load extends beyond on to the partly loaded panel, and the weight on this distance is,

$$\frac{w'}{l}\left(\frac{npl}{l-p} - np\right)$$

$$= \frac{w'}{l}\left(\frac{np^2}{l-p}\right).$$

Of this weight, the part which is borne by the end beyond the load, or the end farthest from the loaded abutment, of that panel into which the load extends, is, by the principles of the lever,

$$\frac{w'}{2pl}\left(\frac{np^2}{l-p}\right)^2. \quad - \; - \; - \; - \; - \; - \; - \; - \quad (34)$$

This amount is to be deducted from Eq. (33) to obtain the correct strain upon the brace from the last panel end under the load, towards the unloaded abutment, whence,

$$\frac{w'}{2l^2}\left(\frac{npl}{l-p}\right)^2 - \frac{w'}{2pl}\left(\frac{np^2}{l-p}\right)^2,$$

$$= \frac{w'}{2l^2}\left(\frac{n^2p^2l^2 - n^2p^3l}{(l-p)^2}\right),$$

$$= \frac{w'n^2p^2l}{2l^2(l-p)}.$$

$$\because \quad V = \frac{w'n^2p^2}{2l(l-p)}, \quad - \; - \; - \; - \; - \; - \; - \quad (35)$$

is the greatest vertical strain from a partial load, where n represents the number of panels loaded, towards the unloaded abutment upon the brace at the last loaded panel point.

Example.—Let a truss be 80 feet long, divided into 8 panels, and loaded at the rate of one ton per foot:

What is the greatest vertical strain upon the brace from the last loaded panel end, when six panels are loaded?

$$\begin{aligned} \text{Here, } w' &= 80 \text{ tons,} \\ l &= 80 \text{ feet,} \\ p &= 10 \quad \text{"} \\ n &= \;\;6 \quad \text{"} \end{aligned}$$

Substituting these values in Eq. (35) we have

$$V = \frac{w'n^2p^2}{2l(l-p)} = \frac{80 \times 6^2 \times 10^2}{2 \times 80(80-10)} = 25.71 \text{ tons.}$$

If in $\frac{w'}{2l^2}(l-u)^2$, $(l-u)$ had been made equal to 65, we should have $\frac{80}{2.80^2}(65)^2 = 26.41$ tons for the reaction of the unloaded abutment.

But as the load extends half way from one panel end to the other, the unloaded panel end would support of $\frac{1}{4}$ of $\frac{1}{2}$ panel load, or 1.25 ton; hence

$26.41 - 1.25 = 25.16$ tons, would be the greatest strain upon the brace from the sixth panel end.

In the case supposed, $\frac{npl}{l-p} = \frac{6 \times 10 \times 80}{80-10} = 68.57$ feet, is the length of the load, and consequently, 29.38 tons is the reaction of the unloaded abutment; and 3.67 tons the weight on the first unloaded panel end, whence,

$$29.38 - 3.67 = 25.71 \text{ tons,}$$

as before, for the greatest strain on the brace.

When np is greater than $\frac{l}{2}$, or when the load covers more than half the truss, $\frac{w'n^2p^2}{2l(l-p)}$ is passing in the same direction as the vertical strain at the same point from the constant truss load, and consequently, Eq. (35) may be added to Eq. (18), giving

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'n^2p^2}{2l(l-p)} \quad - - - - \quad (36)$$

for the maximum vertical strain, from both the permanent and the movable loads, in the braces of that panel whose centre is distant u from the unloaded abutment, and where n is the number of panel points in $l-u$. A reference to Fig. (14) will show that $n = \dfrac{l-u-\frac{p}{2}}{p}$; substituting this value in Eq. (36) we obtain,

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'\left(l-u-\frac{p}{2}\right)^2}{2l(l-p)}, \quad - - \quad (37)$$

an equation giving the same results and containing only one variable quantity, u.

But when np of Eq. (35) is less than $\frac{l}{2}$, then the vertical strain, $\frac{w'n^2p^2}{2l(l-p)}$, is passing towards the centre or in an opposite direction to $\frac{w}{2} - \frac{wu}{l}$, the vertical strain from the constant truss weight at the same point, and the difference between the two is the total vertical strain at u. The less of these may be considered as neutralizing its amount in the greater; but $\frac{w}{2} - \frac{wu}{l}$, the constant truss strain, also meets the vertical strain from the first panel point outside the load, or the quantity, $\frac{w'}{2pl}\left(\frac{np^2}{l-p}\right)^2$, Eq. (34), and has been lessened by this amount outside the load. Hence, since $\frac{w'n^2p^2}{2(l-p)^2}$ Eq. (33) and $\frac{w}{2} - \frac{wu}{l}$ are each diminished by the same amount, their difference will remain the same, and we have

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'n^2p^2}{2(l-p)^2} \quad - \; - \; - \; - \quad (38)$$

for vertical strains to the farther abutment when the truss is less than half loaded.

Substituting for n its value $\frac{l-u-\frac{p}{2}}{p}$, Eq. (38) becomes

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'\left(l-u-\frac{p}{2}\right)}{2(l-p)^2}, \quad - \; - \; - \quad (39)$$

the vertical strain to the unloaded abutment when the load covers less than half the truss affecting the brace at the last loaded panel end.

CHAPTER III.

A SIMPLE TRUSS, WITH INCLINED STRUTS AND VERTICAL TIES SUBJECT TO THE ACTION OF A CONSTANT AND A MOVING LOAD.

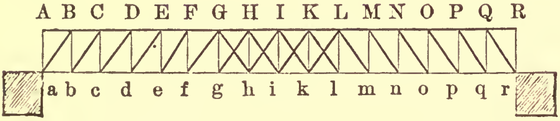


Fig. 18.

55.—Let w = 150,000 lbs., the weight of the truss, uniformly distributed,

w' = 300,000 lbs., the weight of the full moving load, of equal density with the partial load,

l = 200 feet, the length of the truss,

d = 18.75 feet, the depth of the truss,

p = 12.5 feet, the length of a panel,

x = the distance of the end of a panel from one abutment,

u = the distance of the centre of a panel from one abutment,

H. V. & L. = the horizontal, vertical, and longitudinal strains.

The moving load upon the lower chord, and the weight of the truss, may, with sufficient practical accuracy, be considered as concentrated upon the panel points of the same chord.

For the horizontal strains which are greatest when the truss is fully loaded, we have, Eq. (25),

$$H = \frac{(w+w')x}{2d} - \frac{(w+w')x^2}{2dl},$$

and substituting the values of the constants,

$$H = \frac{(150{,}000 + 300{,}000)x}{2 \times 18.75} - \frac{(150{,}000 + 300{,}000)x^2}{2 \times 18.75 \times 200}$$
$$= 12{,}000x - 60x^2.$$

Any value of x in this case gives the compression in the upper chord on the centre side of the point to which x is measured, and the tension in the lower chord on the abutment side; whence we have the following table of strains:

Values of x.	12.5	25	37.5	50	62.5	75	87.5	100
Strains in lbs.	140,625	262,500	365,625	450,000	515,625	562,500	590,625	600,000
Compression in.	BC & PQ	CD & OP	DE & NO	EF & MN	FG & LM	GH & KL	HI & IK	
Tension in.	ab & qr	bc & pq	cd & op	de & no	ef & mn	fg & lm	gh & kl	hi & ik

The vertical strain in any tie is the same in amount as that in the strut to the upper end of which it is attached, (35).

For the maximum vertical strains when the truss is more than half loaded, we have, (Eq. 37),

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'\left(l - u - \frac{p}{2}\right)^2}{2l(l-p)}$$

and substituting the values of the constants,

$$V = \frac{150,000}{2} - \frac{150,000u}{200} + \frac{300,000(200 - u - 6.25)^2}{2 \times 200(200 - 12.5)}$$
$$= 75,000 - 750u + 4.(193.75 - u)^2$$

For the maximum vertical strains when the truss is less than half loaded, we have, Eq. (39),

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'\left(l - u - \frac{p}{2}\right)^2}{2(l-p)^2}.$$

Substituting values,

$$V = 75,000 - 750 + 4.267(193.75 - u)^2$$

Beginning with the truss fully loaded, we will consider the load as gradually moved off, making the first value of u, or the length of the unloaded part, 6.25 feet, the second value of u, 18.75 feet, and so on from either end. Whence the following table of tension in the ties:

Values of u.	6.25	18.75	31.25	43.75	56.25	68.75	81.25	93.75
Strains in lbs.	210,938	183,438	157,188	132,188	108,438	85,938	64,688	64,688
Tension in.	Bb & Qq	Cc & Pp.	Dd & Oo	Ee & Nn.	Ff & Mm	Gg & Ll	Hh & Kk	Ii

When the truss is less than half loaded some of the ties act as counterties, or to carry the weight towards the centre; but the strain thus brought upon them is less than that to which they are subject when the load reaches from them to the farther abutment.

The vertical strain multiplied by the length of the strut and divided by the depth of the truss, or, in this case, $V \times 1.202$, gives the compression in the struts. As long as V has a plus value, it indicates a strain towards the unloaded abutment.

The following is a table of the compression in the struts:

Values of u.	6.25	18.75	31.25	43.75	56.25	68.75	81.25	93.75	106.25	118.75
Strains in lbs.	253,547	220,492	188,939	158,889	130,342	103,297	77,754	53,714	33,631	11,945
Compression in	Ba & Qr	Cb & Pq	Dc & Op	Ed & No	Fe & Mn	Gf & Lm	Hg & Kl	Ih & Ik	Hi & Ki	Gh & Lk

The next value of u, 131.75, gives V a minus value or a strain passing to the loaded abutment; therefore, Gh, Hi, Ki, and Lk, are the counterbraces needed in this case.

There are no strains in AB, Aa, QR, and Rr.

56.—There are many practical disadvantages in the use of trusses, where the ties are shorter than the struts; and as the equations for determining strains in trusses with vertical struts and inclined or diagonal ties are the same, and can be readily applied to the former case, we shall confine our further investigations to the latter.

CHAPTER IV.

TRUSSES WITH VERTICAL STRUTS AND INCLINED TIES SUBJECT TO CONSTANT AND TO MOVING LOADS.

CASE I.—A SIMPLE TRUSS.

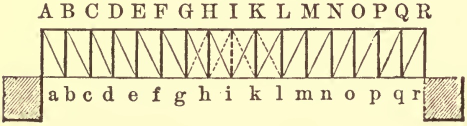


Fig. 19.

57.—Let w = 40 tons, the weight of the truss, uniformly distributed,

w' = 80 tons, the weight of a full load of equal density with the partial load,

l = 80 feet, the length of the truss,

d = 10 feet, the depth of the truss,

p = 5 feet, the length of a panel,

$\left.\begin{matrix}\text{H. V. L.}\\ x \,\&\, u\end{matrix}\right\}$ = the strains and distances, as before.

The load is upon the upper chord, and consequently the struts have the same vertical strains as the ties to the lower ends of which they are attached.

58. Horizontal Strains.—In investigating the horizontal strains, the counterbraces shown in the figure by the dotted lines may be considered as removed, to prevent confusion in deciding to which side of the panel points, or ends, the equation of horizontal strain is to be applied; as the total horizontal strain at any point is in that member of the chord on the side of the panel point on which there is no brace.

Eq. (25),

$$H = \frac{(w+w')x}{2d} - \frac{(w+w')x^2}{2dl},$$

will give the maximum compressions in the upper-chord members on the nearest abutment sides of the points to which x is measured, and the maximum tensions in the lower-chord members on the centre or opposite sides of the same points.

Substituting the values of the above constants, we have,

$$H = \frac{(40+80)x}{2 \times 10} - \frac{(40+80)x^2}{2 \times 10 \times 80} = 6x - .075x^2,$$

whence the following table:

Values of x.	5	10	15	20	25	30	35	40
Strains in Tons.	28.1	52.5	73.1	90.	103.31	112.5	118.1	120
Compression in	AB & QR	BC & PQ	CD & OP	DE & NO	EF & MN	FG & LM	GH & KL	HI & IK
Tension in	bc & pq	cd & op	de & no	ef & mn	fg & lm	gh & kl	hi & ik	

59. Vertical Strains.—Substituting the values given in Eq. (37), we have,

$$V = \frac{40}{2} - \frac{40 \times u}{80} + \frac{88\left(80 - u - \frac{5}{2}\right)^2}{2 \times 80(80 - 5)} = 20 - \frac{u}{2} + \frac{(77.5 - u)^2}{150},$$

whence the following table of compression in the struts when the truss is more than half loaded. Aa and Rr each evidently bear half the load, or the whole vertical strain which comes upon the abutment.

Values of u.		2.5	7.5	12.5	17.5	22.5	27.5	42.5	37.5
Strains in Tons.	60	56.3	48.9	41.9	35.3	28.9	22.9	17.6	17.6
Compression in	Aa & Rr	Bb & Qq	Cc & Pp	Dd & Oo	Ee & Nn	Ff & Mm	Gg & Ll	Hh & Kk	Iii

Since V of Eq. (37), multiplied by the length of the ties, and divided by d, which is $V \times 1.118$, when the load covers more than half the truss; and of Eq. (39), which, with the constants substituted, is

$$V = 20 - \frac{u}{2} + \frac{(77.5 - u)^2}{140.625}$$

multiplied also by 1.118, when the load covers less than half the truss, gives the tension in the ties, we have the following table:

Values of u	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.4
Strains in Tons.	62.9	54.7	46.8	39.5	32.3	25.6	19.7	13.3	8.3	3.0
Tension in	Ab & Rq	Bc & Qp	Cd & Po	De & On	Ef & Nm	Fg & Ml	Gh & Lk	Hi & Ki	Ik & Ih	Kl & Hg

When $u = 52.5$, V has a minus value, or all the counterbraces needed in this truss, are Ih, Kl, and Hg, Ik, and the vertical Ii.

CASE II.—A DOUBLE TRUSS WITH AN EVEN NUMBER OF PANELS.

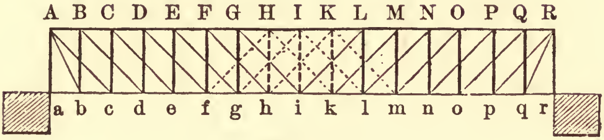


Fig. 20.

60.—This truss, Fig. 20, is a combination of two simple trusses, one of which is represented in Fig. 21, with the counterbraces removed, and which is divided into panels of uniform length.

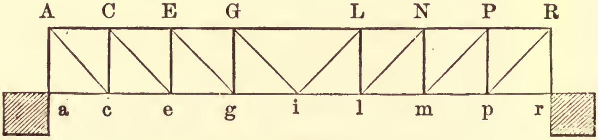


Fig. 21.

The other simple truss, represented in Fig. 22, also without the counterbraces, has all the panels of the same length as in Fig. 21, except the end panels which are of half the length.

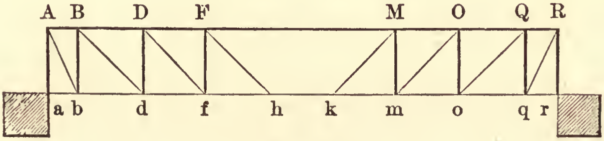


Fig. 22.

The counterbraces are the dotted lines in Fig. 20.

The vertical strains in the simple trusses are entirely independent of each other, for there is no connection between their braces. The chords, however, are common, and the strains upon them in the double truss are the sums of the chord strains of the simple trusses.

61. Horizontal Strains.—The strain in MN, for example, in Fig. 20, is the sum of the strain in LN of Fig. 21, and the strain in MO, of Fig. 22. Hence we have to determine the strains in the simple trusses, and add them, to obtain the strains in the double truss. Each truss may be properly considered as bearing half the weight, and the reaction of either abutment is therefore $\frac{w}{4}$, upon each of the simple trusses.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel of the double truss,
w = the weight upon the truss, uniformly distributed,
x = the distance to a panel end from one abutment,
H = the horizontal strain,
V = the vertical strain.

For the simple truss of Fig. 21, we have therefore Eq. (14), w being changed to $\frac{w}{2}$.

$$H = \frac{wx}{4d} - \frac{wx^2}{4dl}. \qquad - \quad - \quad - \qquad (40)$$

This equation will not apply to the other simple truss, on account of its half panel at the ends. The uniform simple truss has a full panel load at each panel point, and a half panel load upon each abutment; the other simple truss has a full panel load at each panel point, and none upon the abutment. To obtain an equation for the latter, we have $\frac{w}{4} \times x'$ for the moment of the reaction of the abutment, at any panel point distant x' from that abutment; the load upon the truss between this point and the abutment is $\frac{w}{2l} + (x'-p)$, p being the length of the end panel; and the distance of its centre of gravity is $\frac{x}{2} + \frac{p}{2}$; this is apparent from the figure of the truss. Whence,

$$H' = \frac{wx'}{4d} - \frac{w}{2dl}(x'-p)\left(\frac{x'+p}{2}\right) = \frac{wx'}{4d} - \frac{wx'^2}{4dl} + \frac{wp^2}{4dl} \quad - \quad - \quad - \quad - \quad - \quad (41)$$

is the compression in the upper chord, and tension in the lower chord of the truss of Fig. 22 at any point distant x' from the abutment.

If x in Eq. (40) be equal to MR in Fig. 22, H will be the horizontal strain in MO and km; and if x' of Eq. (41) be equal to LR in Fig. 21, H′ will be the horizontal strain in LN and il.

Hence, the horizontal strain in the upper chord of the double truss at any panel point, is equal to the horizontal strain in that one of the simple trusses whose panel end is at the same point, added to the strain at that panel end of the other simple truss which comes next towards the centre; or H at x in Fig. 20 is equal to H at x in one of the simple trusses, added to H′ at $x + p$ in the other simple truss.

Making, therefore, x' of Eq. (41) equal to $x + p$, and adding the equation so changed to Eq. (40), we have,*

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl} - \frac{pwx}{2dl} + \frac{pw}{4d},$$

$$= \frac{w}{2d}\left(x + \frac{p}{2}\right) - \frac{w}{2dl}\left(x + \frac{p}{2}\right)^2 + \frac{wp^2}{8dl}. \quad (42)$$

For the panel points of the double truss, common also to simple truss, Fig. 22, the strains at x' are equal to the strains at x' in one simple truss added to the strains in the other simple truss at $x' + p$, making, therefore, x of Eq. (40) equal to $x' + p$, and adding the equation so changed to Eq. (41), we obtain the same result as before; or Eq. (42) gives the strains in all the members of the upper chord on the nearest abutment side of the panel points to which x is measured.

In the lower chord, the strain at any point of the double truss is equal to the strain in one simple truss at the same point added to the strain in the other simple truss at the next panel point nearer the abutment; making, therefore, x' of Eq. (41) $= x' - p$, or x of Eq. (40) $= x - p$ and adding the equation so changed to the other unchanged, the result from either addition is,

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl} + \frac{pwx}{2dl} - \frac{pw}{4d},$$

$$= \frac{w}{2d}\left(x - \frac{p}{2}\right) - \frac{w}{2dl}\left(x - \frac{p}{2}\right)^2 + \frac{wp^2}{8dl}, \quad (43)$$

* The algebraic process, being very simple, is omitted.

an equation giving the tensions in the lower-chord members, on the centre sides of the points to which x is measured.

62. Vertical Strains.—The simple trusses being entirely independent of each other in their vertical actions, the equations of their vertical strains are to be deduced from the simple truss horizontal strains of Eqs. (40) and (41) as in (32), whence we obtain, from either Eq.

$$V = \frac{w}{4} - \frac{wu}{2l}, \qquad (44)$$

for the vertical strain from a full load in either simple truss, u being the distance to the centre of a panel of a simple truss, and not to the centre of a panel of the compound truss. In the simple truss of Fig. 22, the centre of the end panel is considered as at the abutment, and the first value of u for that truss is therefore zero.

63. Vertical Strains from the Moving Load.—The effect of the moving load upon the panel points of a compound truss differs from that upon the points of a single truss, because, in the former case, a panel point or end of one simple truss can be fully loaded without the next panel point belonging to the same simple truss being affected by any portion of the load. So that the effects of the load in a compound truss are the same as if the different portions of it, or panel loads, were suspended at the ends of the panels.

Therefore, w' representing the weight of the full movable load,

$$V = \frac{w'}{4l^2}(l - u)^2,$$

will give the greatest vertical strain from the moving load, w', upon the simple truss of Fig. 22.

Hence, adding this to Eq. (44),

$$V = \frac{w}{4} - \frac{wu}{2l} + \frac{w'}{4l^2}(l-u)^2 \quad - \; - \; - \quad (45)$$

is the equation of the vertical strain from the constant load, w, and the moving load, w', in simple truss Fig. 22. In the simple truss of Fig. 21, where u' is the distance from the unloaded abutment to the centre of one of the panels, $\frac{w'}{2l}(l-u'-p)$, as an inspection of the figure will show, is the load on $l-u'$; dividing by l and multiplying by $\frac{l-u'+p}{2}$, the distance of its centre of gravity from the abutment from which the load extends, and we have,

$$V = \frac{w'}{4l^2}((l-u')-p^2), \quad - \; - \; - \; - \quad (46)$$

for the vertical strain upon the truss from the moving load. Adding this to the equation for the constant load, we have,

$$V = \frac{w}{4} - \frac{wu'}{2l} + \frac{w'}{4l^2}((l-u')^2-p^2) \quad - \; - \quad (47)$$

for the equation of the vertical strain from the constant and the moving loads in simple truss Fig. 21.

64. EXAMPLE.—In Fig. 20,

Let $w' =$ 160 tons, the weight of the full moving load,

$w =$ 80 tons, the weight of the truss,

$l =$ 160 feet, the length of the truss,

$d =$ 20 feet, the depth of the truss,

$p =$ 10 feet, the length of a panel,

$x =$ the distance from the abutment to the end of a panel,

$u =$ distance from the abutment to the centre of a panel of either of the simple trusses.

The load is upon the lower chord.

Substituting the values of these constants in Eq. (42), we have,

$$H = \frac{(w'+w)}{2d}\left(x+\frac{p}{2}\right) - \frac{(w'+w)}{2dl}\left(x+\frac{p}{2}\right)^2 + \frac{(w'+w)p^2}{8dl},$$

$$= \frac{(160+80)}{2\times 20}\left(x+\frac{10}{2}\right) - \frac{(160+80)}{2\times 20\times 160}\left(x+\frac{10}{2}\right)^2 + \frac{(160+80)10^2}{8\times 20\times 160} = 6(x+5) - .0375\,(x+5)^2 + .9375.$$

from which we can form the following table of compressions in the members of the upper chord:

Values of x.	10	20	30	40	50	60	70	80
Strains in Tons.	82.5	127.5	165	195	217.5	232.5	240	240
Compression in	AB & QR	BC & PQ	CD & OP	DE & NO	EF & MN	FG & LM	GH & KL	HI & IK

Substituting the values of the constants in Eq. (43), we have,

$$H = 6(x-5) - .0375\,(x-5)^2 + .9375,$$

whence we can form the following table of tensions in the lower chord:

Values of x.	10	20	30	40	50	60	70
Strains in Tons.	30	82.5	127.5	165	195	217.5	232.5
Tension in	bc & pq	cd & op	de & no	ef & mn	fg & lm	gh & kl	hi & ik

There are no strains in ab and qr.

Substituting the values of the constants in Eq. (45), we have,

$$V = \frac{80}{4} - \frac{80u}{2 \times 160} + \frac{160}{4 \times (160)^2}(160-u)^2$$

$$= 20 - .25u + \frac{(160-u)^2}{640}.$$

We can form the following table of compressions in the struts of simple truss of Fig. 22:

Values of u.	0	20	40	60	80
Strains in Tons.	60	45.6	32.5	20.6	10
Compression in	Aa & Rr	Bb & Qq	Dd & Oo	Ff & Mm	Hh & Kk

Dividing the same Eq. by d and multiplying by the length of the tie, or V $\times$ 1.414, for all ties except the end ones, where it is, V $\times$ 1.118, and we obtain the following table of tensions in the ties of Fig. 22. (The quantities by which V is multiplied are the secants of the angles made by the ties with a vertical line.)

Values of u.	0	20	40	60	80	100
Strains in Tons.	67.1	64.5	46	29.1	14.1	0.9
Tension in	Ab & Rq	Bd & Qo	Df & Om	Fh & Mk	Hk & Kh	Km & Hf

V has a negative value when $u = 110$.

Substituting constants in Eq. (47), we have,

$$V = 20 - .25u + \frac{(160 - u')^2 - 100}{640},$$

Whence the following table of compressions in the struts of simple truss of Fig. 21:

Values of u'.	10	30	50	70
Strains in Tons.	52.5	38.8	26.3	15
Compression in	Aa & Rr	Cc & Pp	Ee & Nn	Gg & Ll

The total compression in the end struts, Aa and Rr, from the two simple trusses is $52.5 + 60 = 112.5$ tons.

Multiplying V of Eq. (47) by 1,414, as before, we have the following table of tensions in the ties of truss Fig. 21:

Values of u'.	10	30	50	70	90
Strains in Tons.	74.2	54.9	37.2	21.2	7.1
Tension in	Ac & Rp	Ce & Pn	Eg & Nl	Gi & Li	Il & Ig

65.—The double truss shown in Fig. 20 has an even number of panels, and each half of it has also an even number. If two panels be added, each half of the truss will then contain an odd number of panels, and simple truss of Fig. 22 will be uniformly divided, while the ends of simple truss of Fig. 21 will become similar to the ends of the other simple truss in the example given. Each truss, however, will still support one-half the full load, and the horizontal equations will remain unchanged.

The vertical equations for the moving load, however, are entirely dependent upon the end panels of the simple trusses, and equation (45) will apply to that truss which is divided into uniform panels, that is, terminates with a panel equal to two panels of the compound truss; while vertical equation (47) will always apply to that simple truss whose end panel is equal to one panel of the compound truss.

66.—By referring to the examples given, it will be seen that the strains in the two chords are equal in amount between the same inclined braces.

CASE III.—A DOUBLE TRUSS CONTAINING AN ODD NUMBER OF PANELS.

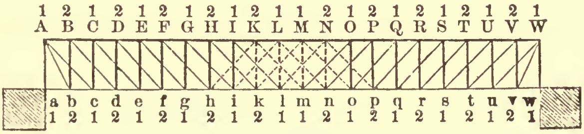


Fig. 23.

67.—Let Fig. 23 represent a double truss, the counterbracing shown by the dotted lines, containing an odd number of panels.

This truss is also composed of two simple trusses, the panel points of one being shown by the figures 1,1,1, and the panel points of the other by the figures 2,2,2. The two simple trusses will be distinguished as truss No. 1 and truss No. 2, as they are numbered in the Figure.

Simple truss No. 1, having full end panels, may be considered as bearing the half panel loads resting directly on the abutments, and has consequently one panel load more than the other; or it supports half the whole weight and half a panel load; while Truss No. 2 supports half the whole weight, less half a panel load. This is when the truss is fully loaded.

68.—Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel of the double truss,
w = the weight upon the truss, uniformly distributed,
x = the distances to the panel ends from one abutment,
H = the horizontal strain,
V = the vertical strain.

69. Horizontal Strain.—The weight on simple truss No. 1 being $\frac{w}{2} + \frac{wp}{2l}$, the reaction of each abutment is therefore $\frac{1}{2}\left(\frac{w}{2} + \frac{wp}{2l}\right)$, and as the truss is divided into uniform panels, the moment of the load on any segment whose length is x, is $\frac{1}{2}\left(\frac{wx^2}{2l}\right)$; from which we readily obtain the equation,

$$Hd = \tfrac{1}{2}\left(\frac{w}{2} + \frac{wp}{2l}\right)x - \frac{wx^2}{4l},$$

whence,

$$H = \frac{wx}{4d} - \frac{wx^2}{4dl} + \frac{wpx}{4dl}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

is the compression in the upper chord of Simple Truss No. 1 on the abutment side, from which x is measured, of the points 1,1,1, &c., and the tension in the lower chord of the same truss on the centre side of the same points.

The reaction of each abutment upon Simple Truss No. 2 is $\frac{1}{2}\left(\frac{w}{2} - \frac{wp}{2l}\right)$, and the moment of the load on any segment whose length is x, is, as in the Simple Truss of Fig. 22,

$$\tfrac{1}{2}\left(\frac{wx'^2}{2l} - \frac{wp^2}{2l}\right),$$

from which we obtain the equation,

$$Hd = \tfrac{1}{2}\left(\frac{w}{2} - \frac{wp}{2l}\right)x' - \tfrac{1}{2}\left(\frac{wx'^2}{2l} - \frac{wp}{2l}\right),$$

whence,

$$H = \frac{wx'}{4d} - \frac{wx'^2}{4dl} - \frac{wpx'}{4dl} + \frac{wp^2}{4dl} \quad - \quad - \quad (49)$$

is the compression in the upper chord of Simple Truss No. 2 on the side towards that abutment from which x' is measured, of the points 2,2,2, &c., and the tension in the lower chord on the centre side of the same points.

It is evident that here, as in the previous case, in the upper chord of the double truss, the compression at any panel point is the compression at the same point of one of the simple trusses added to the compression in the other simple truss at one panel length nearer the centre; and similarly in the lower chord of the double truss, the tension at any panel point is equal to the tension at the same point of one of the simple trusses, added to the tension in the other simple truss at one panel length nearer the abutment.

Therefore, making x' of Eq. (49) equal to $x + p$, and adding the equation so changed to Eq. (48), we have,

$$\mathrm{H} = \frac{wx}{2d} - \frac{wx^2}{2dl} + \frac{wp}{4d} - \frac{wpx}{2dl} - \frac{wp^2}{4dl}$$

$$= \frac{w}{2d}\left(x + \frac{p}{2}\right) - \frac{w}{2dl}\left(x + \frac{p}{2}\right)^2 - \frac{wp^2}{8dl}, \quad (50)$$

for the compression in the members of the upper chord on the abutment side of the points 1, 1, &c.

And making x of Eq. (48) equal to $x' + p$, and adding to (Eq. (49), we have,

$$\mathrm{H} = \frac{wx'}{2d} + \frac{wp}{4d} - \frac{wx'^2}{2dl} - \frac{wpx'}{2dl} + \frac{wp^2}{4dl}$$

$$= \frac{w}{2d}\left(x' + \frac{p}{2}\right) - \frac{w}{2dl}\left(x' + \frac{p}{2}\right)^2 + \frac{3wp^2}{8dl}, \quad (51)$$

for the compression in the members of the upper chord on the abutment side of the points 2, 2, 2, &c.

In these and the subsequent equations x cannot have a value greater than $\frac{l}{2}$, because the simple trusses are not symmetrical, as in the previous cases, beyond the centre.

Making x' of Eq. (49) equal to $x-p$ of Eq. (48), and adding the two, we have,

$$H = \frac{wx}{2d} - \frac{wp}{4d} - \frac{wx^2}{2dl} + \frac{wpx}{2dl} + \frac{wp^2}{4dl},$$

$$= \frac{w}{2d}\left(x - \frac{p}{2}\right) - \frac{w}{2dl}\left(x - \frac{p}{2}\right)^2 + \frac{3wp^2}{8dl}, \quad - \quad (52)$$

for the tension in the members of the lower chord of the double truss on the centre side of the points of Simple Truss No. 1.

And making x of Eq. (48) equal to $x'-p$ of Eq. (49) and adding the two, we have,

$$H = \frac{wx'}{2d} - \frac{wp}{4d} - \frac{wx'^2}{2dl} + \frac{wpx'}{2dl} + \frac{wp^2}{4dl},$$

$$= \frac{w}{2d}\left(x' - \frac{p}{2}\right) - \frac{w}{2dl}\left(x' - \frac{p}{2}\right)^2 - \frac{wp^2}{8dl}, \quad - \quad (53)$$

for the tension in the members of the lower chord of the double truss on the centre side of the points of Simple Truss No. 2.

70.—Let there be an odd number of panels on either side of the centre panel as in Fig. 24,

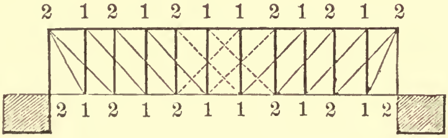


Fig. 24.

and numbering the trusses as before, Simple Truss No. 1 has now a half panel at either end, while the end panels of Simple Truss No. 2 are uniform with the others. Truss No. 1 still supports one panel load more than Truss No. 2, but the moment of the load on the segment x is

$$\frac{wx^2}{4l} - \frac{wp^2}{4l},$$

whence,

$$H = \frac{wx}{4d} + \frac{wpx}{4dl} - \frac{wx^2}{4dl} + \frac{wp^2}{4dl}, \quad - \quad - \quad - \qquad (54)$$

as the horizontal strain in Simple Truss No. 1.

The moment of the load on segment x' of Simple Truss No. 2 is $\frac{wx'^2}{4dl}$, whence,

$$H = \frac{wx'}{4d} - \frac{wx'^2}{4dl} - \frac{wpx'}{4dl}. \quad - \quad - \quad - \quad - \quad - \quad - \qquad (55)$$

Following the same process as before, substituting and adding, we obtain the same results, that is, Eq. (50) for compression in the upper chord at the points 1, 1, &c.; Eq. (51) for compression in the same chord at the points 2, 2, &c.; Eq. (52) for tension in the lower chord at the points 1, 1, &c., and Eq. (53) for tension in the same chord at the points 2, 2, &c.

Hence we see that these equations are not affected by the panels at the ends of the simple trusses, but that Eqs. (50) and (52) belong to the panel points of that simple truss whose braces form the centre panel, and Eqs. (51) and (53) belong to the points of the other simple truss

71.—It will lead to less confusion, therefore, to measure the points from the centre of the truss, as well as render the equations simpler; putting in Eq. (50), $\frac{l}{2} - z$ for x; z being the distance from the centre of the truss to the same point to which x is measured, we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 - \frac{wp^2}{8dl}, \quad - \; - \; - \; - \quad (56)$$

for upper-chord compressions at the panel points of Simple Truss No. 1.

In Eq. (51), putting $\frac{l}{2} - z'$ for x', we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{p}{2}\right)^2 + \frac{3wp^2}{8dl}, \quad - \; - \; - \quad (57)$$

for upper-chord compressions at the panel points of Simple Truss No. 2.

In Eq. (52), putting $\frac{l}{2} - z$ for x, we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z + \frac{p}{2}\right)^2 + \frac{3wp^2}{8dl}, \quad - \; - \; - \quad (58)$$

for lower-chord tensions at the panel points of simple Truss No. 1.

In Eq. (50), putting $\frac{l}{2} - z'$ for x', we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' + \frac{p}{2}\right)^2 - \frac{wp^2}{8dl}, \quad - \; - \; - \; - \quad (59)$$

for lower-chord tensions at the panel points of Simple Truss No. 2.

72. Vertical Strain from the Constant Load.—Under a uniformly distributed load, the vertical strains in one simple truss are unaffected by those in the other, and the equations are therefore to be deduced, as before, from the simple-truss horizontal equations. (32)

From Eq. (48) or Eq. (54) we obtain for Simple Truss No. 1,

$$V = \frac{w}{4} - \frac{wu}{2l} + \frac{wp}{4l}. \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad (60)$$

From Eq. (49) or Eq. (55), for Simple Truss No. 2,

$$V = \frac{w}{4} - \frac{wu'}{2l} - \frac{wp}{4l}, \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad (61)$$

u and u' being the distances to the centres of the panels of the simple trusses, and whenever either simple truss begins with a half panel, u or u' in the equation which belongs to that truss must be made equal to zero. It will be noticed that the constant-load vertical equations, like the compound-truss horizontal equations, are unaffected by the terminations of the simple trusses; but are determined in their application to either simple truss by the position of the braces of that truss at the centre.

73.—If the difference in the successive values of u and u' be constant when these quantities exceed $\frac{l}{2}$, each will then represent the distances to the centres of the panels of the other simple truss than that for which the equation in which it is found was obtained. That is, $\begin{Bmatrix} u \\ u' \end{Bmatrix}$ of Eq. $\begin{Bmatrix} 60 \\ 61 \end{Bmatrix}$, which, when less than $\frac{l}{2}$, is the distance from one abutment to the centre of any panel point of Simple Truss $\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$ becomes, when greater than $\frac{l}{2}$, the distance from the same abutment to the centre of any panel of Simple Truss $\left\{\begin{matrix}2\\1\end{matrix}\right\}$.

Further, when u or u' becomes greater than $\frac{l}{2}$, the equation to which it belongs gives the vertical strain in the other simple truss, or that one to the centres of whose panels it now represents the distances; that is Eq. $\left\{\begin{matrix}60\\61\end{matrix}\right\}$, when $\left\{\begin{matrix}u\\u'\end{matrix}\right\}$ is greater than $\frac{l}{2}$, gives the vertical strain in Simple Truss No. $\left\{\begin{matrix}2\\1\end{matrix}\right\}$ passing to the abutment opposite that from which $\left\{\begin{matrix}u\\u'\end{matrix}\right\}$ is measured. For Eq. (60),

$$V = \frac{w}{4} - \frac{w}{2l}\left(u - \frac{p}{2}\right],$$

becomes, when u is greater than $\frac{l}{2}$, and consequently equal to $l - u'$,

$$V = -\left\{\frac{w}{4} - \frac{w}{2l}\left(u' + \frac{p}{2}\right]\right\},$$

or the same as Eq. (61) when u' is measured, as indicated by the minus sign, from the opposite abutment; and similarly is Eq. (61) changed.

74. Vertical Strains from the Moving Load.—It will be seen from the plan of the truss, Fig. 23, that the passing load, before reaching the centre, transmits that portion of its weight which is borne by the farther abutment through the counterbraces from one of the simple trusses to the other. One half of one simple truss being thus connected with the opposite half of the other, we have two other simple trusses in this same double truss, different from the former simple trusses, and in their vertical action under a moving load entirely independent of each other; they are shown in Figs. 25 and 26.

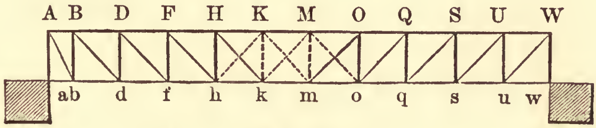


Fig. 25.

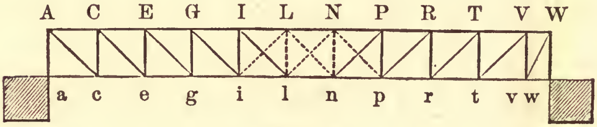


Fig. 26.

Either of these is simply the other reversed. In this case, as well as in any combination of simple trusses, any number of panels, in either simple truss, extending from one end may be considered as fully loaded without throwing any weight upon the panel point outside the load as in the case of the Simple Truss (54).

Let the moving load extend any distance from the $\left\{\begin{matrix}\text{left}\\ \text{right}\end{matrix}\right\}$ end of truss of Fig. $\left\{\begin{matrix}25\\ 26\end{matrix}\right\}$, or that abutment on which a half panel end of the simple truss rests,

then,
$$V = \frac{w'(l-u)^2}{4l^2}$$

is the reaction of the opposite abutment, and the greatest vertical strain at any point from the moving load passing to the unloaded abutment, w' being the weight of the full movable load.

But if the load extend from the opposite abutment, or that on which a full panel end of one of these trusses rests; then Eq. (45),

$$V = \frac{w'}{4l^2}\left((l-u)^2 - p^2\right),$$

will give the reaction of the opposite abutment and the greatest vertical strain.

These equations depend for their application upon the length of the end panel of the simple truss upon which the load enters, and are not affected by the length of the panel at the other end of this simple truss. And since either simple truss may have a full or half panel, as the double truss contains more or less, either of the moving-load equations may be added to either of the simple-truss constant-load equations. There is no difficulty, however, in determining how the addition is to be made in any case.

If the moving load extend from the half panel end of the trusses of Fig. 25 and Fig. 26, and cover more than half the truss, then it is plain that $\frac{w'(l-u)^2}{4l^2}$ must be added to the equation of that simple truss which has a full panel end, since it is the braces of this truss that transmit the strain $\frac{w'(l-u)^2}{4l^2}$ to the unloaded abutment; and if the load covers less than half the truss, then $\frac{w'(l-u)^2}{4l^2}$ is acting upon the braces of the other simple truss between the end of the load and the centre of the truss. Hence we have this simple rule:

$\dfrac{w'(l-u)^2}{4l^2}$ is to be added to the equation of that simple truss which has full panel ends.

Similarly it may be shown that $\dfrac{w'}{4l^2}[(l-u)^2-p^2]$ is to be added to the equation of that simple truss which has half panel ends.

75. Example of the Application of the Vertical Equations.—

In Fig. 23, let l = 210 feet, the length of the truss,
d = 20 feet, the depth of the truss,
p = 10 feet, the length of a panel,
w = 105 tons, the weight of the truss,
w' = 210 tons, the weight of a full movable load.

Since Simple Truss No. 1 has full panel ends, we have,

$$V = \frac{w'(l-u)^2}{4l^2} + \frac{w}{4} - \frac{w}{2l}\left(u - \frac{p}{2}\right) \quad - \; - \; - \quad (62)$$

for the maximum vertical strains in the braces of this simple truss, and in the counterbraces of Simple Truss No. 2.

Substituting the values of the constants in Eq. (62) we have,

$$V = \frac{210(210-u)^2}{4 \times (210)^2} + \frac{105}{4} - \frac{105}{2 \times 210}\left(u - \frac{10}{2}\right),$$

$$= \frac{(210-u)^2}{840} + 26.25 - 0.25\,(u-5).$$

This gives the maximum strains in the struts. For the tension in the ties, Eq. (62), must be multiplied by 1.414, the secant of the angle made by the ties with the verticals; whence we can form the following table:

Values of u.	10	30	50	70	90	110	130
Strains in Tons.	75	58.6	45.5	33.3	22.1	11.9	2.6
Compression in	Ww & Aa	Uu & Cc	Ss & Ee	Qq & Gg	Oo & Ii	Mm & Ll	Kk & Nr
Strains in Tons.	106.1	82.9	64.3	47.1	31.2	16.8	3.7
Tension in	Wu & Ac	Us & Ce	Sq & Eg	Qo & Gi	Om & Il	Mk & Ln	Kh & Np

The strains in the end braces, that is, the end strut and the ties attached to it, in this and in the next table, are determined as follows: It will be seen by referring to the double truss with an even number of panels, that the strain on the end ties of either simple truss is the same as if it had been obtained from the constant-load vertical equation, with $w' + w$ substituted for w, or the same as where the truss is fully loaded. This is because the simple trusses are entirely independent of each other, in their vertical action either under a full or a partial load. But in the present case, when the truss is partially loaded, one half of one simple truss is connected by the counterbraces to the opposite half of the other simple truss, and strains come, more or less, upon these counterbraces, until the opposite halves of the same simple truss are fully loaded, then the counterbraces are released, and the moving-load equation is no longer applicable. Hence, in this case, the greatest strains upon the end braces, which are when the truss is fully loaded, are to be determined from the simple-truss constant-load vertical Eqs. (60) and (61), w of these equations being changed to $w' + w$.

Since Simple Truss No. 2 has half panel ends, we have,

$$V = \frac{w'}{4l^2}[(l-u')^2-p^2] + \frac{w}{4} - \frac{w}{2l}(w'+p), \quad (63)$$

for the maximum vertical strains in all the braces of this simple truss, and in the counterbraces of Simple Truss No. 1.

Substituting the constants in Eq. (63) we have,

$$V = \frac{210}{4\times(210)^2}[(210-u')^2-(10)^2] + \frac{105}{4} - \frac{105}{2\times 210}\left(u' + \frac{10}{2}\right),$$

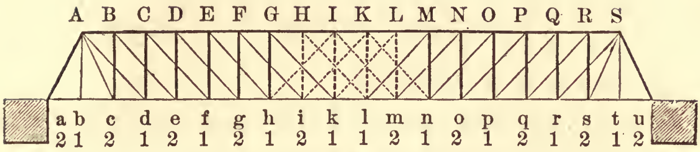
$$= \frac{(210-u')^2 - 100}{840} + 26.25 - 0.25(u'+5).$$

This gives the maximum strains in the struts; for the strains in the ties, we must multiply Eq. (63) by 1.414, as before, except for the end ties, when it is multiplied by 1.118 (the secant of their angle); whence we can form the following table:

Values of u'.	0	20	40	60	80	100	120
Strains in Tons.	75	62.9	49.3	36.7	25	14.4	4.5
Compression in	Ww & Aa	Vv & Bb	Tt & Dd	Rr & Ff	Pp & Hh	Nn & Kk	Ll & Mm
Strains in Tons.	83.9	88.9	67.7	51.9	35.4	20.2	6.4
Tension in	Wv & Ab	Vt & Ba	Tr & Df	Rp & Fh	Pn & Hk	Nl & Km	Li & Mo.

It will be observed that the struts Ll and Mm are common to both trusses, and are subjected to a greater strain by Eq. (62), and that Kk and Nn are also common, but subjected to greater by Eq. (63), or when the longer segment is loaded. The compression upon the end struts Aa and Ww, from the passing load, 75+75 = 150 tons, the sum of the strains in the simple trusses.

76. The Quincy Railroad Bridge.—The longest span of this bridge offers an excellent opportunity for the application of the preceding formulæ, as it is divided into an odd number of panels, with end panels differing from the others, and probably presents as complicated an example of the double truss as is likely to occur.



QUINCY RAILROAD BRIDGE.

Fig. 27.

* Let $l =$ 247 feet, the length of the truss,
$d =$ 26 feet, the depth of the truss,
$p =$ 13 feet, the length of a panel,
$w =$ 198,150 lbs., the weight of the truss, uniformly distributed.
$w' =$ 328,750 lbs., the weight of the uniform full movable load.

In this case, the panel points of Simple Truss No. 1, or the simple truss whose braces form the centre panel, are K, l, M, n, O, p, Q, r, S and t, to the right of the centre; and to the left, I, k, G, h, E, f, C, d, A and b; the remaining points are those of Simple Truss No. 2. In the upper chord, the uniformity of the double truss extends from A to S, and in the lower chord, from c to s; Eqs. (56, 57, 58, 59) will therefore apply between those points, for the horizontal strains; from a to c and from s to u, the horizontal tension is readily found from the moment of the reaction of the abutment around A or S. This tension being greatest under a full load, where $\frac{w'+w}{2}$ is the reaction of the abutment, whence we have, deducting half panel load on the abutment,

* These dimensions and weights are from the description of the Quincy Bridge by Mr. T. C. Clarke, C. E.

$$H = \frac{wp}{2d} - \frac{wp^2}{2dl}, = \frac{(w'+w)p}{2d} - \frac{(w'+w)p^2}{2dl}$$

for the strain in a c and s u.

Substituting the values of the constants in Eq. (56), w being equal to $w' + w$, we have,

$$H = \frac{(328,750+198,150)\times 247}{8\times 26} - \frac{328,750+198,150}{2\times 26\times 247}$$
$$\left(z - \frac{13}{2}\right)^2 - \frac{(328,750+198,150)13^2}{8\times 26\times 247}$$
$$= 623,961 - 41.023(z - 6.5)^2,$$

Here, z is the distance from the centre to the panel points, and as this equation belongs to Simple Truss No. 1, the different values of z are 6.5, 32.5, 58.5, 84.5, and the amount of compression, given by the substitution of these different values of z in the equation, is contained in that member of the upper chord on the abutment side of the points to which z is measured.

Substituting the values of the constants in Eq. (57) we have,

$$H = 630,894 - 41.023(z - 6.5)^2:$$

z is here the distance from the centre to the panel points of Simple Truss No. 2, and its values are consequently 19.5, 45.5, 71.5, and 97.5, by the substitution of which, in the equation, we obtain the compression in the upper-chord members on the abutment side of the point to which z is measured.

Eq. (58) becomes

$$H = 630,894 - 41.023\ (z + 6.5)^2.$$

tension in the lower chord on the centre side of panel points of Simple Truss No. 1, values of z being 6.5, 32.5, 58.5, and 84.5; and Eq. (59) becomes

$$H = 653{,}961 - 41.023\,(z + 6.5)^2,$$

tension in the lower chord on the centre side of panel points of Simple Truss No. 2, the values of z being 19.5 45.5, 71.5, and 97.5.

The strains in the lower chord are the same in amount as those in the upper chord between the same inclined braces, and consequently the lower-cord equations are only needed in this case to obtain the strain in c d and r s.

From these equations, by the substitution of the different values of z, we can form the following table of strains in the chords

Values of z.	6.5	19.5	32.5	45.5	58.5	71.5	84.5	97.5	
Strains in lbs.	623961	623961	596229	568498	513035	457571	374377	291182	
Compression in	HI, IK & KL	GH & LM	FG & MN	EF & NO	DE & OP	CD & PQ	BC & QR	AB & RS	
Strains in lbs.	623961	596229	568498	513035	457571	374377	291182	180256	124792
Tension in	kl	ik & lm	hi & mn	gh & no	fg & op	ef & pq	de & qr	cd & rs	ab, bc, st & tu

The load is upon the lower chord, consequently the struts have the same vertical strain as those ties to the upper ends of which they are attached.

In the figure, the counterbraces needed under the effects of the moving load are shown by the dotted lines. In the bridge itself there are counterbraces from every point in the lower chord, except the points b and t; why the reason which compels the insertion of these superfluous counterbraces is not applicable to these two points it is impossible to say.

When the moving load covers part of the truss, b and t may be considered as belonging to Simple Truss No. 1, for the other might be removed and this one would support these points.

In which case this simple truss, then, has end panels of half the length of the others; that is, it has a panel point distant from the abutment half the length of one of its panels; and being connected with Simple Truss No. 2 at the centre by the counterbraces, the maximum vertical strain in the braces of the latter, when u is less than $\frac{l}{2}$, and in the counterbraces of No. 1, when u is greater than $\frac{l}{2}$, is, from the moving load,

$$V = \frac{w'}{4l^2}(l - u)^2,$$

u, as before, being the distance from the abutment to the centre of the panel in which are the braces whose strains are to be determined; and adding to Eq. (61), we have for the total vertical strain,

$$V = \frac{w'}{4l^2}(l - u)^2 + \frac{w}{4} - \frac{w}{2l}\left(u + \frac{p}{2}\right). \quad - \quad - \quad (64)$$

The part of the moving load borne by the points b and t may be considered as belonging to either simple truss; since the distance of one of these points is p from the nearest abutment, when it is loaded, the reaction of the farther abutment is plainly (8) $\frac{w'p^2}{l^2}$. When the load enters Simple Truss No. 2, which has uniform end panels, the vertical moving-load strain from it is Eq. (45), $\frac{w'}{4l^2}[(l-u')^2-p^2]$; to which add $\frac{w'p^2}{l^2}$ and constant-load strain, Eq. (60), and we have,

$$V = \frac{w'}{4l^2}[(l-u')^2-p^2]+\frac{w}{4}-\frac{w}{2l}\left(u'-\frac{p}{2}\right), \quad - \quad (65)$$

for the total maximum vertical strains in the braces of No. 1 when u' is less than $\frac{l}{2}$, and in the counterbraces of No. 2 when greater than $\frac{l}{2}$.

The ambiguity in regard to the load on the points b and t renders it necessary to provide in one simple truss for a slight excess of strain. This arises from the fact that the symmetry of the truss is broken at these points.

Substituting the values of the constants in Eq. (64) we have,

$$V = \frac{328{,}750}{4\times(247)^2}(247-u)^2+\frac{198{,}150}{4}-\frac{198{,}150}{2\times 247}(u+6.5),$$
$$= 1.347(247-u)^2+49{,}537.5-401.1(u+6.5),$$

compression in the struts of Simple Truss No. 2, and vertical component of the tension in the ties. $V \times 1.414$

gives the longitudinal strain of the latter. Whence we have the following table of strains in the braces of this truss :

Values of u.	13	39	65	91	117	143
Strains in lbs.	124,016	89,564	65,477	43,211	22,765	
Compression in	Su & Aa	Rs & Bc	Pq & De	No & Eg	Lm & Hi	
Strains in lbs.	124,016	126,643	95,584	61,100	32,189	5,858
Tension in	Ss & Ac	Rq & Be	Po & Dg	Nm & Fi	Lk & Hi	Ih & Kn

The compression in Aa and Su, and the tension in Ac & Ss is obtained from constant-load equation, w made equal to $w'+w$, and multiplied by 1.118, the secant of the angle of these two braces. When $u=130$, the strain in Ik and Kl is less than when the opposite segment of the truss is loaded.

Substituting the values of the constants in Eq. (65), we have,

$$\begin{aligned}\mathrm{V} &= \frac{328{,}750}{4\times(247)^2}[(247-u)^2+3(13^2)]+\frac{198{,}150}{4} \\ &\quad -\frac{198{,}150}{2\times247}(u-6.5) \\ &= 1.347\,(l-u)^2+50{,}220-401.1\,(u-6.5),\end{aligned}$$

compression in the struts of Simple Truss No. 1, and vertical component of the tension in the ties; $\mathrm{V}\times 1.414$ gives the longitudinal strain in the latter, the strain in the end braces from the constant-load equations, as before; whence we have the following table of strains in the braces of this truss:

Values of u.	26	52	78	104	130
Strains in lbs.	124,016	83,190	60,013	38,658	19,122
Compression in	Su & Aa	Qr & Cd	Op & Ef	Mn & Gh	Kl & Ik
Strains in lbs.	156,850	117,631	84,858	54,662	26,028
Tension in	Sr & Ad	Qp & Cf	On & Eh	Gk & Ml	Ki & Im

As the compression upon Su and Aa comes from both simple trusses, we have for its total amount, $124{,}016+124{,}016+27{,}732\times 1.118 = 279{,}035$ lbs.

The tension upon Ab and St is the weight of a full panel load, 27,732 lbs.

All the counterbraces needed in this truss are, Ih, Im, Ki, Kn, Lk, Hl, Hi, Ik, Kl, and Lm.

CASE IV.—A TRIPLE TRUSS CONTAINING AN EVEN NUMBER OF PANELS.

77.—Let Fig. 28 represent a triple truss divided into an even number of panels.

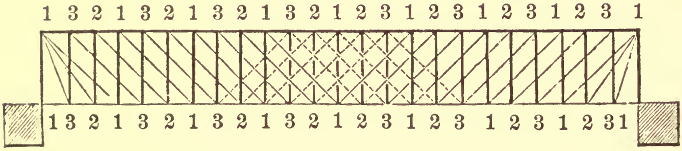


Fig. 28.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel,
w = the weight supported by the truss, uniformly distributed,
$x, x'\ x''$ = the distances from one abutment to the panel ends,
u &c. = $x + \frac{3p}{2}$, &c.
H & V = the horizontal and vertical strains.

78. Horizontal Strains.—Under the maximum uniform load, in which case the horizontal strains are the greatest, this truss may be considered as divided into three simple trusses whose vertical strains, or strains having vertical components, do not in any way affect each other.

These simple trusses, with the counterbracing shown

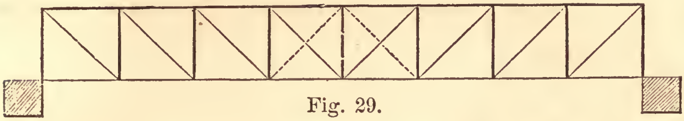


Fig. 29.

by the dotted lines in Fig. 28 removed, are, Truss No. 1, Fig. 29, whose braces meet at the centre; Truss

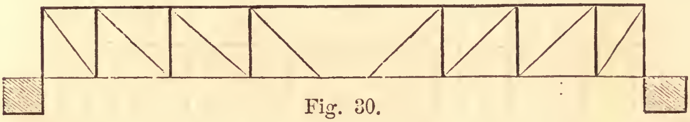


Fig. 30.

No. 2, Fig. 30, whose braces come next; and Truss No. 3, Fig. 31.

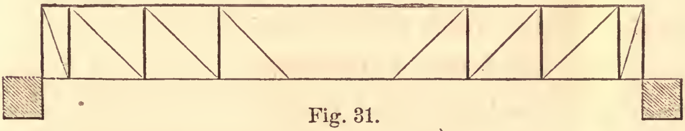


Fig. 31.

The regular panels of these simple trusses are equal to three panels of the triple truss in length; Simple Truss No. 1 is uniform throughout, and having a full panel length at the end may be considered as supporting the half panel loads resting upon the abutments. The end panels of Simple Truss No. 2 are two-thirds, and the end panels of Simple Truss No. 3 one-third the length of their other panels; and the symmetry of both is broken at the centre.

Under the uniform full load each simple truss bears one-third the weight, or $\frac{w}{3}$; the reaction of each abutment upon Simple Truss No. 1, is therefore $\frac{w}{6}$, the weight upon a segment of the truss, $\frac{1}{3}$ of $\frac{wx^2}{2l}$ x being the distance from the abutment to the panel points of the truss hence,

$$H = \frac{wx}{6d} - \frac{wx^2}{6dl} \quad - \quad - \quad - \quad - \quad - \quad (66)$$

is the horizontal strain in the upper and lower chords of Simple Truss No. 1.

In simple Truss No. 2, the reaction of the abutment is $\frac{w}{6}$, the weight upon any segment of the truss is $\frac{1}{3}$ of $\frac{w}{l}(x' - 2p)$, (x' being the distance from the abutment to a panel point of this truss, and confined to values less than $\frac{l}{2}$), and the distance of the centre of gravity of this weight from the point to which x' is measured, is $\frac{x'}{2} + \frac{p}{2}$; whence we have, for the moment of the load on x', $\frac{wx'^2}{6l} - \frac{wpx'}{6l} - \frac{wp^2}{3l}$; therefore,

$$H = \frac{wx'}{6d} - \frac{wx'^2}{6dl} + \frac{wpx'}{6dl} + \frac{wp^2}{3dl} \quad - \quad - \quad - \quad (67)$$

is the horizontal strain in the upper and lower chords of Simple Truss No. 2.

Similarly, in Simple Truss No. 3, the reaction of each abutment is $\frac{w}{6}$, the load on x'' is $\frac{1}{3}$ of $\frac{w}{l}(x'' - p)$, (x'' being the distance from the abutment to a panel point of this truss, and confined to values less than $\frac{l}{2}$), and the distance of its centre of gravity from the point to which x'' is measured is $\frac{x''}{2} + p$, whence we have for the moment of the load on x'', $\frac{wx''^2}{6l} + \frac{wpx''}{6l} - \frac{wp^2}{3l}$;

Therefore,

$$H = \frac{wx''}{6d} - \frac{wx''^2}{6dl} - \frac{wpx''}{6dl} + \frac{wp^2}{3dl} \quad - \; - \; - \quad (68)$$

is the horizontal strain in the upper and lower chords of Simple Truss No. 3.

These equations give the strains in the simple trusses on the abutment sides in the upper, and on the centre sides in the lower chord, of the points to which x, x' and x'' are measured; the strain in the upper chord of the triple truss, at any panel point, is the strain in that simple truss whose panel point is at the same place, added to the simple-truss strains at the next two panel points towards the centre; and, in like manner, the strain in the lower chord of the triple truss, at any panel point, is the strain in one simple truss at the same point, added to the simple-truss strains at the next two panel points towards the abutment. Hence, to find the compression in the triple truss at the panel points of Simple Truss No. 1, we must make x'' of Eq. (68) equal to $x + p$, x' of Eq. (67) equal to $x + 2p$, and add the equations so changed to Eq. (66); performing this operation, we obtain,

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl} + \frac{wp}{2d} - \frac{wpx}{dl}. \quad - \; - \; - \; - \quad (69)$$

For the compression in the triple truss, at the panel points of Simple Truss No. 2, make x of Eq. (66) equal to $x'+p$, x''' of Eq. (68) equal to $x'+2p$, and then, adding to Eq. (67), we have,

$$\mathrm{H} = \frac{wx'}{2d} - \frac{wx'^2}{2dl} + \frac{wp}{2d} - \frac{wpx'}{dl} - \frac{wp^2}{2dl}. \quad - \quad - \quad (70)$$

For the compression at the panel points of Simple Truss No. 3, in the triple truss, make x' of Eq. (67) equal to $x''+p$, x of Eq. (66) equal to $x+2p$, and, adding to Eq. (68), we have,

$$\mathrm{H} = \frac{wx''}{2d} - \frac{wx''^2}{2dl} + \frac{wp}{2d} - \frac{wpx''}{dl}, \quad - \quad - \quad - \quad (71)$$

the same as Eq. (69); or, but two equations are required for the upper chord, one, Eq. (69), for the points of Simple Trusses No. 1 and No. 3, and one, Eq. (70), for the points of Simple Truss No. 2.

In the lower chord, by a similar process, and substituting $-p$ and $-2p$ for p and $2p$, we obtain one equation,

$$\mathrm{H} = \frac{wx}{2d} - \frac{wx^2}{2dl} - \frac{wp}{2d} + \frac{wpx}{dl}, \quad - \quad - \quad - \quad - \quad (72)$$

for the tension in the triple truss at the panel points of both Simple Trusses, Nos. 1 and 2, and,

$$\mathrm{H} = \frac{wx''}{2d} - \frac{wx''^2}{2dl} - \frac{wp}{2d} + \frac{wpx''}{dl} - \frac{wp^2}{2dl}, \quad - \quad (73)$$

for the tension in the triple truss at the panel points of Simple Truss No. 3.

These equations, giving the maximum strains in the chords of the truss of Fig. 28, will remain the same so long as the truss contains an even number of panels, and are not affected by the end panels of the simple trusses. That is to say, if, in the case supposed, another panel be added at either end, though Simple truss No. 1 will then support $\frac{1}{3}\left(w - \frac{2wp}{l}\right) + \frac{wp}{l}$; Simple Truss No. 2, the same, and Simple Truss No. 3, $\frac{1}{3}\left(w - \frac{2wp}{l}\right)$, the same equations will still apply; the trusses being numbered, as in this case, from the centre. It must be remembered that the moment of the load on any simple truss is not affected by the proportion of the full load which it bears, but depends upon the length of the end panel of that simple truss. For example, if this truss be lengthened two panels, Simple Truss No. 1 bears $\frac{w}{3} - \frac{wp}{3}$, but the moment of the load upon it is, because its end panel is one-third the length of its other panels, the same as the moment of the load upon Simple Truss No. 3, in the example above, which has a panel of equal length, or,

$$\frac{wx^2}{6dl} + \frac{wpx}{6dl} - \frac{wp^2}{3dl}$$

This is because the centre of gravity is affected by the length of the end panel.

79. Strain in the Second Lower-Chord Members.— In the lower chord, as the equation applies to the abutment end of any member, it is evident that, to obtain the strain in the second member from the abutment, we require the simple-truss strain for the point next the abutment, added only to the simple-truss strain at the panel point on the abutment, as the latter is the only panel point on the abutment side which may be added to the former. But making x, in the equation of the latter simple truss, zero, this strain becomes zero, and we need, consequently, only the simple-truss strain at the point one panel length from the abutment. This strain will differ as the point may belong to the different simple trusses, and is simply the reaction of the abutment upon that simple truss to which the point referred to belongs, multiplied by p and divided by d.

If the point belongs to Simple Truss No. 1, the reaction of the abutment is always,

$$\tfrac{1}{6}\left(w+\frac{wp}{l}\right),$$

whence we have,

$$H = \frac{wp}{6d} + \frac{wp^2}{6dl}, \quad - \; - \; - \quad (74)$$

for the strain in the second or next to end member of the lower chord.

If the point belongs to Simple Trnss No. 2, the reaction of the abutment is always,

$$\tfrac{1}{6}\left(w - \frac{wp}{l}\right),$$

whence we have,

$$H = \frac{wp}{6d} - \frac{wp^2}{6dl}, \quad - \; - \quad (75)$$

for the strain in the second member. And if the point belongs to Simple Truss No. 3, the reaction of the abutment is always $\frac{w}{6}$, whence we have,

$$H = \frac{wp}{6d}, \quad - \; - \; - \; - \; - \; - \quad (76)$$

for the strain in the second member.

There is no strain in the end members of the lower chord, and the strains in all the other members may be obtained from the regular equations given above.

If the inclined braces were struts, instead of ties, a similar process would determine the strains in the corresponding members of the upper chord.

80. Vertical Strains from the Constant Load.—The vertical equations are obtained from the simple-truss horizontal equations, as in the previous cases, because, under a full load, the simple-truss braces are unconnected, and act independently of each other, and the difference in the horizontal strains in any simple truss at x and at $x+3p$, or at the two ends of a panel, is the horizontal component of the strain in the inclined brace connecting these two points, and, as before, the vertical component may be obtained from the proportion, $3p : d$.

In Simple Truss No. 1, if the end panel be of uniform length with the others, or equal to $3p$,

$$H = \frac{wx}{6d} - \frac{wx^2}{6dl},$$

is the horizontal strain; if the end panel $= 2p$, it becomes

$$H = \frac{wx}{6d} - \frac{wx^2}{6dl} + \frac{wp^2}{3dl},$$

and if the end panel $= p$, it is

$$H = \frac{wx}{6d} - \frac{wx^2}{6dl} + \frac{wp^2}{3dl};$$

from either of which equations we obtain, as before,

$$V = \frac{w}{6} - \frac{wu}{3l}, \quad - \quad - \quad - \quad - \quad (77)$$

for the vertical strain from a constant load in Simple Truss No. 1, u being equal to $x + \frac{3p}{2}$, or the distance to the centre of a panel of the simple truss. Whence we see that the vertical equation is independent of the proportion of the whole weight which the simple truss may sustain.

From Eq. (67) we obtain,

$$V = \frac{w}{6} - \frac{w}{3l}\left(u' - \frac{p}{2}\right), \quad - \quad - \quad - \quad - \quad (78)$$

for the vertical strain in Simple Truss No. 2. And from Eq. (68) we obtain,

$$V = \frac{w}{6} - \frac{w}{3l}\left(u'' + \frac{p}{2}\right), \quad - \quad - \quad - \quad - \quad (79)$$

for the vertical strain in Simple Truss No. 3.

In these equations, u, u', and u'' represent the distances from one abutment to the centres of the panels of the three simple trusses, and in Simple Truss No. 1, the difference in the different consecutive values of u is uniform, or equal to $3p$; this is true of u' and u'' only to the centre of the truss, because the simple trusses to which they belong are not uniform beyond that point. If, in Eq. (79),

u'' be made greater than $\frac{l}{2}$, and the regular increment in its values be kept the same, $3p$, it becomes the distance to the centre of a panel of Simple Truss No. 2, and the values of V will then give the vertical strains in the latter truss passing to or sustained by the abutment opposite to that from which u'' is measured. This may be proved by making $u'' = l - u'$ whence we shall have the equation of Simple Truss No. 2, with the minus sign. Similarly, if u' be made greater than $\frac{l}{2}$, we obtain the vertical strain in Simple Truss No. 3 beyond the centre.

81. Vertical Strains from the Moving Load.—Disregarding the constant load, if we suppose one end of the truss to be loaded, and trace the course of the vertical strain to the other end, we find, in Simple Truss No. 1, that it follows the braces and counterbraces of that truss throughout; but in the other simple trusses, the vertical strain at the centre passes through the counterbraces from one to the other; or, if we consider a part less than one-half of either Simple Truss No. 2 or No. 3 as loaded, the farther abutment reacts upon it through the other simple truss. Hence we have, under the moving load, two other simple trusses, composed of the opposite halves of Simple Trusses Nos. 2 and 3, as shown in Figs. 32 and 33—one being the other reversed.

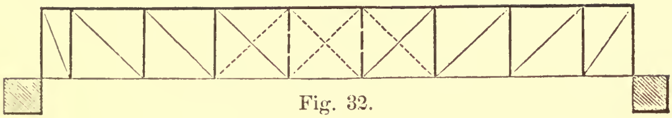


Fig. 32.

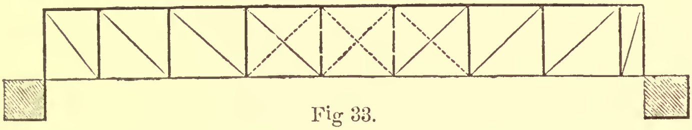


Fig 33.

The dotted lines represent the counterbraces.

In the figure, Simple Truss No. 1 has a panel end equal to $3p$. Let u be the distance from one abutment to the centre of any panel, then $\frac{1}{3}$ of $\frac{w'}{l}\left(l - u - \frac{3p}{2}\right)$, ($w'$ being the weight of the full uniform moving load), will be the weight upon the panel points of this truss within the space $l - u$, and u being unloaded, $\frac{1}{2}\left(l - u + \frac{3p}{2}\right)$ is the distance of the centre of gravity of this weight from the loaded abutment, or the abutment from which the load extends; dividing by l, and multiplying the above quantities (8) we have,

$$V = \frac{w'}{6l^2}\left((l - u)^2 - \frac{9p^2}{4}\right], \quad - \quad - \quad (80)$$

for the reaction of the unloaded abutment and the vertical strain throughout the unloaded part.

Adding Eq. (80) to Eq. (77), we obtain the maximum vertical strain in Simple Truss No. 1, from the moving and the constant loads.

In Fig. 32, which has the left end panel equal to p, and the right end panel equal to $2p$, let u'' be the distance from the right abutment to the centre of any panel, and the length of the unloaded portion of the truss. Then, if the left segment of this truss be loaded, $\frac{1}{3}$ of $\frac{w'}{l}\left(l - u'' + \frac{p}{2}\right)$ will be the weight upon the panel points within the distance $l - u''$, and the distance of the centre of gravity of this load from the left abutment will be $\frac{1}{2}\left(l - u'' - \frac{p}{2}\right)$, whence we have

$$V = \frac{w'}{6l^2}\left((l - u'')^2 - \frac{p^2}{4}\right], \quad - \quad - \quad (81)$$

for the equation of the moving load when the load extends from the left end of Fig. 32, or the right end of Fig. 33, or that end of either truss which has an end panel equal to p, and is to be added, in this case, to the equation of the simple truss at the unloaded end of this truss, or to Eq. (79).

In Fig. 33, which has the left end panel equal to $2p$, and the right end panel equal to p, let u' be the distance from the right abutment to the centre of any panel, and the length of the unloaded portion of the truss; then, if the left segment of the truss be loaded, $\frac{1}{3}$ of $\frac{w'}{l}\left(l - u' - \frac{p}{2}\right]$ will be the weight upon the panel points, within the distance $l - u'$, and the distance of the centre of gravity of this load from the left abutment will be $\frac{1}{2}$ of $\left(l - u' + \frac{p}{2}\right)$, whence we have,

$$V = \frac{w}{6l^2}\left((l - u')^2 - \frac{p^2}{4}\right], \quad - \quad - \quad (82)$$

same as Eq. (81), for the equation of the moving load, when the load extends from left end of Fig. 33 or right end of Fig. 32, or that end of either truss which has a panel equal to $2p$, and is to be added to the equation of the simple truss at the unloaded abutment.

It will be seen that, as the triple truss may vary in the number of its panels, any one may have an end panel equal to p, $2p$, or $3p$, and consequently, either of the equations of the moving load may, in different examples, apply to either of the simple trusses; the application depending solely upon the length of the end panel. The moving-load equation, in any case, is to be added to the constant-load equation of that simple truss through whose braces the reaction of the unloaded abutment acts, to obtain the maximum vertical strain.

82. Horizontal Equations, with the variable measured from the Centre of the Truss.—In Eq. (69) make $x = \frac{l}{2} - z$, and we have,

$$\mathrm{H} = \frac{wl}{8d} - \frac{w}{2dl}(z-p)^2 + \frac{wp^2}{2dl}, \quad - \quad - \quad - \quad (83)$$

for the compression in the upper chord at the panel points of Simple Trusses Nos. 1 and 3.

In Eq. 70, make $x' = \frac{l}{2} - z'$, and we have,

$$\mathrm{H} = \frac{wl}{8d} - \frac{w}{2dl}(z'-p)^2, \quad - \quad - \quad - \quad - \quad - \quad - \quad (84)$$

for the compression in the upper chord at the panel points of Simple Truss No. 2, z and z' being measured from the centre of the truss. By measuring from the centre of the truss, instead of from the ends, the equations are simpler and more readily applied.

83.—Example.

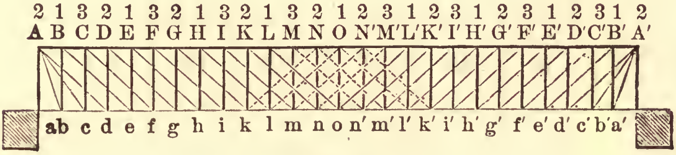


Fig. 34.

Let Fig. 34 represent a triple truss with an even number of panels.

Let $l = 247$ feet, the length of the truss,

$d = 26$ feet, the depth of the truss,

$p = 9.5$ feet, the length of a panel,

$w = 100$ tons, the constant truss weight,

$w' = 160$ tons, the full movable weight.

The load is upon the lower chord. The simple-truss panel points are distinguished in the upper chord by their numbers.

84. Horizontal Strains.—For the upper-chord compressions we have Eqs. (83 and 84), which, with the values of the constants given above, are,

$$H = \frac{260 \times 247}{8 \times 26} - \frac{260}{2 \times 26 \times 247}(z-p)^2 + \frac{260 \times (9.5)^2}{2 \times 26 \times 247},$$

$$= 310.58 - \frac{(z'-p)^2}{49.4},$$

for the points 1 and 3. And

$$H = \frac{260 \times 247}{8 \times 26} - \frac{260}{2 \times 26 \times 247}(z' - p)^2 = 308.75 - \frac{(z'-p)^2}{49.4}.$$

Whence we can form the following table of strains in the upper chord.

Values of z and z'.	0	9.5	19	28.5	38	47.5	57
Strains in Tons.	308.75	308.75	308.75	303.27	292.31	281.35	264.91
Compression in	NO & ON′	MN & N′M′	LM & M′L′	KL & L′K′	IK & K′I′	HI & I′H′	GH & H′G′

(*Continued.*)

Values of z and z'	66.5	76	85.5	95	104.5	114
Strains in Tons.	242.98	221.06	195.66	160.78	127.89	89.53
Compression in	FG & G′F′	EF & F′E′	DE & E′D′	CD & D′C′	BC & C′B′	AB & B′A′

In the lower chord, as the tensions are equal in amount to the compressions in the upper chord, between the same inclined braces, we have only the second and third members from the abutments to determine, the first member being subject to no strain. Since the point one panel length from the abutment belongs to Simple Truss No. 1, we have for the strain in bc and c′b′, Eq. (74),

$$H = \frac{260 \times 9.5}{6 \times 26} + \frac{260 \times (9.5)^2}{6 \times 26 \times 247} = 16.44.$$

and from Eq. (73), making $x'' = 19$, we have the tension in cd and d′c′. We can consequently form the following table without further calculation for the lower-chord tensions:

Strains in Tons.	303.27	292.31	281.35	264.91	242.98	221.06
Tension in	no & o′n′	mn & n′m′	lm & m′l′	kl & l′k′	ik & k′ i′	hi & i′h′

(*Continued.*)

Strains in Tons.	195.66	160.78	127.89	89.53	45.67	16.44
Tension in	gh & h′g′	fg & g′f′	ef & f′e′	de & e′d′	cd & d′c′	bc & c′b′

85. Vertical Strains.—Since Simple Truss No. 1 has a panel point distant p from the abutment, Eq. (81) will give the strains from the moving load, and adding this to Eq. (77), we have

$$V = \frac{w'}{6l^2}\left((l - u'')^2 - \frac{p^2}{4}\right) + \frac{w}{6} - \frac{w''u}{3l}. \quad - \quad (85)$$

Substituting the values of the constants, we have,

$$V = \frac{160}{6 \times (247)^2}\left((247 - u'')^2 - \frac{(9.5)^2}{4}\right) + \frac{100}{6}$$
$$- \frac{100u''}{3 \times (247)}$$
$$= \frac{8}{18302.7}\left[(247 - u'')^2 - 22.5625\right] + 16.67$$
$$- \frac{u''}{7.41},$$

The first value of u'' in this equation is — 4.75, because the centre of the first panel of this truss, if it were uniform with the others, would be 4.75 feet back from the abutment; the other values of u'' are the successive distances from the abutment to the centres of the panels; whence we can form the following table of compression in the struts of this simple truss:

Values of u''.	—4.75	23.75	52.25	80.75	109.25	137.75
Strains in Tons.	45.00	35.23	26.18	17.84	10.20	3.28
Compression in	Aa & A'a'	Bb & B'b'	Ee & E'e'	Hh & H'h'	Ll & Ll'	Oo

The next value of u'' will give V a negative value.

The tension in the ties is found by multiplying Eq. (85) by the secants of their angles, which, for the end ties, is 1.065, and for the others, 1.48; whence we can form the following table:

Values of u.	— 4.75	23.75	52.25	80.75	109.25	137.74
Strains in Tons.	47.92	52.14	38.75	26.4	15.1	4.85
Tension in	Ab & A'b'	Be & B'e'	Eh & E'h'	Hl & H'l'	Lo & l'o'	Ol & Ol'

The strains in the end braces are given by Eq. (85) as well as by the constant-load equations, since this simple truss is the same under the moving as under the constant load, but in the other simple trusses it is different; there, as before explained, the strains in the end braces are to be obtained from the constant-load equations alone.

Since Simple Truss No. 2 has an end panel equal to $3p$, we must add Eq. (80) to Simple Truss No. 3 constant-load equation, Eq. (79), for the vertical strains in all the braces of Simple Truss No. 3, and in the counter-braces of Simple Truss No. 2, when u'' is greater than $\frac{l}{2}$; whence we have,

$$V = \frac{w'}{6l^2}\left((l - u'')^2 - \frac{9p^2}{4}\right) + \frac{w}{6} - \frac{w}{3l}\left(u'' + \frac{p}{2}\right), \quad - \text{(86)}$$

Substituting the values of the constants,

$$V = \frac{8}{18302.7}[(247 - u'')^2 - 203.0625] + 16.67$$
$$- \frac{u'' + 4.75}{7.41}.$$

The first value of u'' is 4.75, whence, and the other values of u'', we can form the following table:

Values of u''.	4.75	33.25	61.75	90.25	118.75	147.25
Strains in Tons.	40.00	31.5	22.68	14.57	7.18	0.5
Compression in.	Aa & A′a′	Cc & C′c′	Ff & F′f′	Ii & I′i′	Mm & M′m′	Nn & N′n′

The end braces from the constant-load equations as before, $w' + w$ being put for w.

Multiplying Eq. (86) by the secants of the angles made by the ties, which for the end tie is 1.24, and for the others 1.48, we can form the following table:

Values of u''.	4.75	33.25	61.75	90.25	118.75	147.25
Strains in Tons.	49.60	46.62	33.57	21.56	10.63	0.74
Tension in	Ac & A'c'	Cf & C'f'	Fi & F'i'	Im & I'm'	Mn' & M'n	N'k' & Nk

Finally, since Simple Truss No. 3 has an end panel equal to $2p$, we must add Eq. (82) to Simple Truss No. 2 constant-load equation, Eq. (78) for the vertical strains in all the braces of the latter, and in the counter-braces of the former, when u' is greater than $\frac{l}{2}$; whence we have,

$$V = \frac{w}{6l^2}\left((l - u')^2 - \frac{p^2}{4}\right) + \frac{w}{6} - \frac{w}{3l}\left(u' - \frac{p}{2}\right). \quad - \quad (87)$$

Substituting the values of the constants,

$$V = \frac{8}{18302.7}[(247 - u') - 22.5625] + 16.67$$
$$- \frac{w' - 4.75}{7.41},$$

whence from the different values of u' we can form the following table:

Values of u'.	14.25	42.75	71.25	99.75	128.25
Strains in Tons.	40.00	29.67	21.10	13.23	6.07
Compression in	Aa & A'a'	Dd & D'd'	Gg & G'g'	Kk & K'k'	Nn & N'n'

The compression in Nn and N'n' is greater in this case than in the previous.

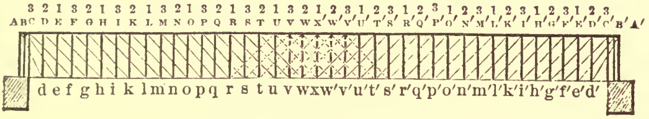
Multiplying Eq. (87) by 1.48, the secant of the angle of the ties, we can form the following table:

Values of u'.	14.25	42.75	71.25	99.75	128.25
Strains in Tons.	59.20	43.93	31.23	19.58	8.98
Tension in	Ad & A'd'	Dg & D'g'	Gk & G'k'	Kn & K n'	Nm' & N'm

The total compression in the end struts is the sum of the compressions in these struts in the above tables, and is consequently 125 tons.

In the example just given the length and depth of the truss, and the weights, are the same as in the example taken from the Quincy Railroad Bridge.

86. Greithausen Bridge.—The large iron railroad bridge (Fig. 35) over the Rhine at Greithausen, is a triple truss containing an even number of panels. It possesses one peculiarity found in some other bridges in Europe, but rarely, if ever, occurring in this country. This peculiarity is, that each of the simple trusses has a separate end strut, two of which are placed back from the edge of the abutment.



GREITHAUSEN BRIDGE.

Fig. 35.

This arrangement, shown in the figure, possesses but slight mechanical advantages, if any, over an end strut common to the three simple trusses; and whatever these may be, they must quickly disappear when we consider that any increase of length in a truss adds to the horizontal strain in both chords throughout their lengths, though the weight remain the same. For, in the Eq. (15)

$$H = \frac{wl}{8d},$$

H increases directly as l; w and d remaining constant, and in Eq. (14),

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl}.$$

H is increased at any point by any increase in the value of l.

By this elongation of the length of the end panels, the moment of the reaction of the support upon two of the simple trusses is increased, but the reaction of the support itself is unaffected, as well as the moment of the load upon any segment, x. No general equation can be given for a case of this kind, but a special equation must be obtained for each simple truss that may be elongated.

Let $\frac{w}{n}$ be the reaction of the support upon any simple truss; then, if this support is at the abutment, at the distance of x from the abutment, the moment will be $\frac{wx}{n}$; but if the support be removed backward a distance a from the abutment, the moment will be $\frac{w}{n}(x+a)$. The moment of the load on x remains the same, and we have for the difference between the two cases a constant, $\frac{wa}{n}$; this divided by d, the depth of the truss, is an increased amount of strain to be added to the horizontal equations.

87. Horizontal Strains.—The dimensions and weights of a truss of this bridge are as follows, the latter being assumed:

$l =$ 320 feet, the distance between the abutments,
$d =$ 24 feet, the depth of the truss,
$p =$ 8 feet, the length of a panel,
$a =$ 2 feet, the length of each of the two small end panels,
$w' =$ 250 tons, the weight of the full moving load,
$w =$ 130 tons, the constant truss weight.

Simple Truss No. 3 is not extended. Simple Truss No. 2 is extended $2a$, its load is $\frac{w}{3}+\frac{2wp}{3l}$; the moment of the reaction of its support is therefore $\left(\frac{w}{3} - \frac{wp}{3l}\right)$ $(x + 2a)$, and the excess, owing to the elongation of the end panel, is $\frac{wa}{3} + \frac{2wpa}{3l}$. Simple Truss No. 1 is extended a; its load is $\frac{w}{3} - \frac{wp}{3l}$; the moment of its support is $\left(\frac{w}{6} - \frac{wp}{6l}\right)(x + a)$, the excess from the lengthened end panel, $\frac{wa}{6} - \frac{wpa}{6l}$; adding these two quantities, and dividing by d, we have,

$$\frac{wa}{2d} + \frac{wpa}{6dl} \quad - \quad - \quad - \quad - \quad - \qquad (88)$$

for the increase in the compression in the upper chord, between the points B and B′, and the increase in the tension in the lower chord, between the points e and e′. Adding this quantity to Eqs. (83 and 84), and substituting the values of the constants given above, we have for the upper-chord strains,

$$H = 651.14 - .02474(z - 8)^2,$$

for the panel points of Simple Trusses Nos. 1 and 3, and

$$H = 649.56 - .02474(z - 8)^2,$$

for the panel points of Simple Truss No. 2.

The strain in A B and A′ B′ is most readily obtained by multiplying the reaction of the support upon Simple Truss No. 2, given above, by 12, the distance of the last panel point, and dividing by 24, the depth of the truss.

The strains in the lower chord are taken from the strains in the upper chord between the same inclined braces. Hence we form the following table of strains in the chords, $w + w'$ being substituted for w in the equations:

Values of z.	0	8	16	24	32	40	48	56
Strains in Tons.	649.56	649.56	649.56	644.81	635.31	625.81	611.56	592.56
Compression in	WX & XW′	VW & W′V′	UV & V′U′	TU & U′T′	ST & T′S′	RS & S′R′	QR & R′Q′	PQ & Q′P′
Tension in				wx & x′w′	vw & w′v′	uv & v′u′	tu & u′t′	st & t′s′

(*Continued.*)

Values of z.	64	72	80	88	96	104	112
Strains in Tons.	573.56	549.81	521.31	592.81	459.56	421.56	383.56
Compression in	OP & P′O′	NO & O′N′	MN & N′M′	LM & M′L′	KL & L′K′	IK & K′I′	HI & I′H′
Tension in	rs & s′r′	qr & r′q′	pq & q′p′	op & p′o′	no & o′n′	mn & n′m′	lm & m′l′

Continued.

Values of z.	120	128	136	144	152	160	
Strains in Tons.	340.81	293.31	241.81	193.56	136.56	79.56	33.25
Compression in	GH & H′G′	FG & G′F′	EF & F′E′	DE & E′D′	CD & D′C′	BC & C′B′	AB & B′A′
Tension in	kl & l′k′	ik & k′i′	hi & i′h′	gh & h′g′	fg & g′f′	ef & f′e′	de & e′d′

88. Vertical Strains from the Constant Load.—As the Eqs. (77, 78, and 79) represent the weight at any point borne by the simple trusses between that point and the point of no vertical strain, it is evident they are unaffected by the peculiarity of this case, and apply without change.

89. Vertical Strains from the Moving Load.—The vertical strain from the moving load is affected by this change or elongation of the end panels, as is evident when we consider the effect upon a lever, loaded at any other point than the centre, of increasing its length at either or both ends.

The equation of the moving load, being the reaction of the unloaded abutment, depends entirely upon the principles of the lever, and is affected by any arrangement of the simple trusses that alters the position of the centre of gravity of the load on the segment $l - u$, as well as by any elongation of the trusses.

Let the load be supposed to enter Simple Truss No. 1, at the end, then the weight on the segment $l - u$, u being the distance from the abutment beyond the load to the end of the load, is $\frac{1}{3}$ of $\frac{w'}{l}\left(l - u - \frac{p}{2}\right)$; the length of the lever, or Simple Truss No. 1, is $l + 2a$, and the distance of the centre of gravity from the support on which the truss rests at the loaded end is $\frac{1}{2}\left(l - u + \frac{p}{2}\right) + a$; whence we have for the equation of the moving load on this truss,

$$V = \frac{w'}{6l(l+2a)}\left[(l-u)^2 - \frac{p^2}{4} + 2a\left(l-u-\frac{p}{5}\right)\right]. \text{ - } (89)$$

It must be remembered that l is the distance between the abutments, or points c and c′, and not the outside length of the truss; and u the distance from the abutment and not from the end of the truss.

Adding Eq. (88) to Eq. (77), we have the greatest vertical strain in the truss.

Next, let the load be supposed to enter Simple Truss No. 2, at the end; the load on the segment $l - u'$ is $\frac{1}{3}$ of $\frac{w'}{l}\left(l - u' + \frac{p}{2}\right)$; the length of this truss, since Simple Truss No. 2 is added to Simple Truss No. 3, at the centre, is $l + 2a$, and the distance of the centre of gravity from the end of the truss which is loaded, is $\frac{1}{2}\left(l - u' - \frac{p}{2}\right) + 2a$, therefore,

$$V = \frac{w'}{6l(l+2a)}\left[(l-u')^2 - \frac{p^2}{2} + 4a\left(l-u'+\frac{p}{2}\right)\right], \text{ - } (90)$$

which is to be added to Eq. (79) for the total vertical strain. And, lastly, the load on $l - u''$ in Simple Truss No. 3, is $\frac{1}{3}$ of $\frac{w'}{l}\left(l - u'' - \frac{3p}{4}\right)$; length of the truss $l + 2a$; and the distance of the centre of gravity, $\frac{1}{2}\left(l - u' + \frac{3p}{2}\right)$; therefore,

$$V = \frac{w}{6l(l+2a)}\left[(l-u'')^2 - \frac{9p^2}{4}\right]; \quad \text{- - } (91)$$

to be added to Eq. (78) for the total vertical strain.

Substituting the values of the constants in Eq. (89), and Eq. (77) added, we have,

$$\frac{(320-u)^2-4u+1248}{2488}+21.67-\frac{13}{96}u.$$

whence we can form the following table of strains in the struts of Simple Truss No. 1; the load being upon the lower chord:

Values of u.	4	28	52	76	100	124	148	172
Strains in Tons.	61.75	52.61	43.92	35.69	27.92	20.62	13.78	7.4
Compression in	Bb & B'b'	Ee & E'e'	Hh & H'h'	Ll & L'l'	Oo & O'o'	Rr & R'r'	Uu & U'u'	Xx.

Multiplying the vertical strain by 1.414, the secant of the tie angle for all the ties except the end ones, which are to be multiplied by 1.667, we have the following table of tensions in the ties:

Values of u.	4	28	52	76	100	124	148	172	196
Strains in Tons.	102.94	74.39	62.10	50.47	39.48	29.16	19.48	10.46	2.09
Tension in	Be & B'E'	Eh & E'h'	Hl & H'l'	Lo & L'o'	Or & O'r'	Ru & R'u'	Ux & U'x	Xu' & X'u	U'r' & Ur

Substituting the values of the constants in Eqs. (90) and (79) added, we have,

$$\frac{(320-u')^2-8u'+2560}{2488}+21.67-\frac{13}{96}(u'+4).$$

whence we can form the following table of compression in the struts of Simple Truss No. 3, and counterstruts of Simple Truss No. 2. The end braces from the constant-load equation.

Values of u'.	12	36	60	84	108	132	156	180
Strains in Tons.	57.00	49.57	41.00	32.89	25.24	17.56	11.34	5.08
Compression in	Cc & C'c'	Ff & F'f'	Ii & I'i'	Mm & M'm'	Pp & P'p'	Ss & S's'	Vv & V'v'	W'w' & Ww

Multiplying the vertical strain by 1.414, we have the following table of tension in the ties of Simple Truss No. 3, and counterties of Simple Truss No. 2:

Values of u'.	12	36	60	84	108	132	156	180
Strains in Tons.	80.60	70.09	59.97	46.51	34.29	24.83	16.03	7.8
Tension in	Cf & C'f'	Fi & F'i'	Im & I'm'	Mp & M'p'	Ps & P's'	Sv & S'v'	Vw' & V'w	W'l' & Wl.

Substituting the values of the constants in Eqs. (91) and (78) added, we have,

$$\frac{(320-u'')-144}{2488}+21.67-\frac{13}{96}(u''-4),$$

whence we can form the following table of compression in the struts of Simple Truss No. 2, and counterstruts of Simple Truss No. 3. The end braces from the constant-load equations.

Values of u''.	—4	20	44	68	92	116	140	164	188
Strains in Tons.	66.5	55.61	46.81	38.47	30.59	23.17	16.21	9.72	3.69
Compression in	Aa & A'a'	Dd & D'd'	Gg & G'g'	Kk & K'k'	Nn & N'n'	Qq & Q'q'	Tt & T't'	Ww & W'w'	V'v' & Vv

Multiplying the vertical strain for the end ties by 1.118, and for the others by 1.414, we have the following table of tension in the ties of Simple Truss No. 2, and counterties of Simple Truss No. 3:

Values of u''.	—4	20	44	68	92	116	140	164	188
Strains in Tons.	74.35	78.63	66.19	54.4	43.25	32.76	22.92	13.74	5.22
Tension in	Ad & A'd'	Dg & D'g'	Gk & G'k'	Kn & K'n'	Nq & N'q'	Qt & Q't'	Tw & T'w'	Wv' & W'v	V's' & Vs

CASE V.—A TRIPLE TRUSS CONTAINING AN ODD NUMBER OF PANELS.

90.—Let Fig. 36 represent a triple truss composed of three simple trusses, and containing an odd number of panels, the counterbraces being shown by the dotted lines.

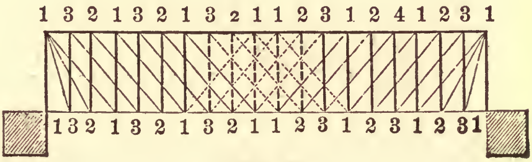


Fig. 36.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel,
w = the weight supported by the truss,
x, x' & x'' = the distances from one abutment to the panel ends,
u, u' & u'' = the distances from one abutment to the centres of the panels of the simple trusses.
H & V = the horizontal and vertical strains.

The simple trusses composing the triple truss are numbered, as in the previous case, from the centre, the panel points belonging to each being shown in the figure by the numbers.

91. Horizontal Strains.—Under the maximum uniform load, w, the weight on simple Truss No. 1, is

$\frac{1}{3}\left(w - \frac{wp}{l}\right) + \frac{wp}{l}$, and the moment around any panel point of this simple truss of the weight on the segment x is, as in the previous case where the simple truss had the same end panel, $\frac{1}{3}$ of $\frac{wx^2}{2dl}$,

$$\therefore \text{H}d = \tfrac{1}{6}\left(w - \frac{wp}{l}\right)x + \frac{wpx}{2l} - \frac{wx^2}{6l},$$

whence,

$$\text{H} = \frac{wx}{6d} - \frac{wx^2}{6dl} + \frac{wpx}{3dl}. \quad - \; - \; - \; - \; - \; - \quad (92)$$

The weight on simple Truss No. 2 is $\frac{1}{3}\left(w - \frac{wp}{l}\right)$, and the moment around any panel point of this simple truss, of the weight on the segment x', is, as before, $\frac{1}{3}\left(\frac{wx'^2}{2l} - \frac{pwx'}{2l} - \frac{wp^2}{2l}\right)$,

whence,

$$\text{H}' = \frac{wx'}{6d} - \frac{wx'^2}{6dl} + \frac{wp^2}{3dl}. \quad - \; - \; - \; - \; - \; - \; - \quad (93)$$

The weight on Simple Truss No. 3 is $\frac{1}{3}\left(w - \frac{wp}{l}\right)$, and the moment around any panel point of this simple truss, of the weight on the segment x'', is, as before, $\frac{1}{3}\left[\frac{wx''^2}{2l} + \frac{wpx''}{2l} - \frac{wp^2}{l}\right]$,

whence,

$$\text{H}'' = \frac{wx''}{6d} - \frac{wx''^2}{6dl} - \frac{wpx}{3dl} + \frac{wp^2}{3dl}. \quad - \; - \; - \quad (94)$$

These are the horizontal strains in the upper and lower chords of the simple trusses at their different panel points. Following the same process which was pursued in the previous case (77), in adding the simple-truss equations, we obtain,

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl} + \frac{wp}{2d} - \frac{wpx}{dl} - \frac{wp}{2dl}. \quad - \quad - \quad (95)$$

Making $x = \frac{l}{2} - z$, we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}(z-p)^2, \quad - \quad - \quad - \quad (96)$$

for the compression in the upper chord of the triple truss at the panel points of the Simple Trusses No. 1 and No. 2.

For the panel points of Simple Truss No. 3 we obtain, by the same process,

$$H = \frac{wl}{8d} - \frac{w}{2dl}(z'-p)^2 - \frac{wp'}{dl}, \quad - \quad - \quad - \quad - \quad (97)$$

z and z' being the distances from the centre of the truss to the panel points of the respective simple trusses.

For the tension in the lower chord of the compound truss, in like manner, we obtain,

$$H = \frac{wl}{8d} - \frac{w}{2dl}(z+p)^2 + \frac{wp^2}{dl}, \quad - \quad - \quad - \quad - \quad (98)$$

for panel points of Simple Truss No. 1, and

$$H = \frac{wl}{8d} - \frac{w}{2dl}(z'+p)^2, \quad - \quad - \quad - \quad (99)$$

for the panel points of Simple Trusses No. 2 and No. 3.

6

These lower-chord equations will not apply to the second panel from the abutment, as that member is only within the action of two of the simple trusses, when x in one of them equals zero, as explained in (78.)

If the point in the lower chord distant p from the abutment belongs to Simple Truss No. 1, the reaction of the abutment upon this truss is always,

$$\frac{w}{3} + \frac{wp}{l}$$

whence,

$$H = \frac{wp}{6d} + \frac{wp^2}{6dl}. \qquad (100)$$

If the point belong to Simple Truss No. 2, the reaction of the abutment upon this truss is always,

$$\tfrac{1}{3}\left(w + \frac{wp}{l}\right),$$

whence,

$$H = \frac{wp}{6d} + \frac{wp^2}{6dl}. \qquad (101)$$

And if the point belong to Simple Truss No. 3, the reaction of the abutment upon this truss is always,

$$\tfrac{1}{3}\left(w - \frac{wp}{l}\right),$$

whence,

$$H - \frac{wp}{6d} - \frac{wp^2}{6dl}. \qquad (102)$$

92. Vertical Strain from a Constant Load.—Deducing the equations of the constant vertical strain from the horizontal equations for the simple trusses, as the vertical strains in the different simple trusses are independent of each other, we have,

$$V = \frac{w}{6} - \frac{w}{3l}(u-p), \quad - \quad - \quad - \qquad (103)$$

for Simple Truss No. 1,

$$V = \frac{w}{6} - \frac{wu'}{3l}, \quad - \quad - \quad - \quad - \quad - \qquad (104)$$

for Simple Truss No. 2, and

$$V = \frac{w}{6} - \frac{w}{3l}(u''+p), \quad - \quad - \quad - \qquad (105)$$

for Simple Truss No. 3.

93. Vertical Strains from the Moving Load.—In this case, Simple Truss No. 2 is always symmetrical at the centre, or its opposite halves are united by the counter-bracing, while one half of Simple Truss No. 1 is connected by the counterbraces with the opposite half of Simple Truss No. 3. This arrangement, however, does not affect the equations for the moving load, and Eqs. (80 and 81) apply in this case as well as in the previous; their application being governed by the same principles.

94.—Example.—Let Fig. 37 represent a triple truss composed of three simple trusses, and containing an odd number of panels; loaded on the lower chord.

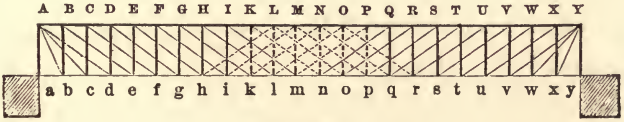


Fig. 37.

Let l = 230 feet, the length of the truss,
d = 20 feet, the depth of the truss,
p = 10 feet, the length of a panel,
w' = 230 tons, the weight of the full moving load,
w = 115 tons, the constant truss weight.

95. Horizontal Strains.—Substituting these values in Eqs. (96) and (97), we have, for upper-chord compressions, w being $(w'+w)$,

$$H = 495.9375 - .0375(z-10)^2,$$

where z is the distance from the centre of the truss to the panel points of Simple Trusses Nos. 1 and 2, and

$$H = 503.4375 - .0375(z'-10)^2,$$

where z' is the distance from the centre of the truss to the panel point of Simple Truss No. 3.

The lower-chord tensions are the same as the upper-chord strains between the same inclined braces, and we shall not need the lower-chord equations except for the third member from the abutment, cd and vw. The strain in these members is the strain at c or w, which are points in Simple Truss No. 1, and consequently given by Eq. (98). As b and x are points in Simple Truss No. 2, the strain in bc and wx is given by Eq. (101.)

Whence we have the following table of strains in the upper and lower chords:

Values of z.	5	15		35	45		65
Values of z'.			25			55	
Strains in Tons.	495	495	495	472.5	450	427.5	382.5
Compression in	LM, MN & NO	KL & OP	IK & PQ	HI & QR	GH & RS	FG & ST	EF & TU
Tension in			mn	lm & no	kl & op	ik & pq	hi & qr

(*Continued.*)

Values of z.	75		95	105	95	
Values of z'.		85				
Strains in Tons.	337.5	292.5	225	157.5	90	30
Compression in	DE & UV	CD & VW	BC & WX	AB & XY		
Tension in	gh & rs	fg & st	ef & tu	de & uv	cd & vw	bc & wx

There is no strain in ab and xy.

96. Vertical Strains.—The moving-load vertical equation depends, as before stated, upon the end panel of the simple truss; and the constant-load vertical equation for any simple truss is to be added to that moving-load equation which applies to that simple truss to which the first is united by the counterbraces at the centre.

Hence, Eq. (103) for Simple Truss No. 1 is to be added to Eq. (80), whence we have,

$$V = \frac{w'}{6l^2}\left((l-u)^2 - \frac{9p^2}{4}\right] + \frac{w}{6} - \frac{w}{3l}(u-p). \quad - \quad - \quad (106)$$

Substituting constants,

$$V = \frac{(230-u)^2 - 225}{1380} + 19.167 - \frac{u-10}{6}.$$

Whence we can form the following table of compressions in the struts. The strains in the end braces from the constant-load equations, as explained before.

Values of u.	5	35	65	95	125
Strains in Tons.	60.00	42.39	29.57	18.04	7.82
Compression in	Aa & Yy	Cc & Ww	Ff & Tt	Ii & Qq	Mm & Nn

Multiplying V by 1.414 for the end tie, and by 1.803 for the others, these being the secants of the tie angles, we obtain the following table of tensions in the ties:

Values of u.	5	35	65	95	125
Strains in Tons.	84.84	76.43	53.31	32.53	14.10
Tension in	Ac & Yw	Cf & Wt	Fi & Tg	Im & Qn	Mp & Nk

For Simple Truss No. 2 we must add Eq. (104) to Eq. (81) and we have,

$$V = \frac{w'}{6l^2}\left((l-u')^2 - \frac{p^2}{4}\right) + \frac{w}{6} - \frac{wu'}{3l}. \quad (107)$$

Substituting constants,

$$V = \frac{(230 - u')^2 - 25}{1380} + 19.167 - \frac{u'}{6},$$

whence we can form the following table of compression in the struts:

Values of u'.	—5	25	55	85	115
Strains in Tons.	60	45.43	32.17	20.22	9.57
Compression in	Aa & Yy	Bb & Xx	Ee & Uu	Hh & Rr	Ll & Oo

Multiplying V by the secants of the tie angles, 1.118 for the first, and 1.803 for the others, we can form the following table of strains in the ties:

Values of u'.	— 5	25	55	85	115	145
Strains in Tons.	67.08	81.99	58.01	36.46	17.26	0.40
Tension in	Ab & Yx	Be & Xu	Eh & Ur	Hl & Ro	Lo & Ol	Or & Lh

Simple Truss No. 2 is the same under the moving and under the constant load, hence Eq. (107) gives the strains in the end braces.

For Simple Truss No. 3, Eq. (103) is to be added to Eq. (80), whence we have,

$$V = \frac{w'}{6l^2}\left((l-u'')^2 - \frac{p^2}{4}\right) + \frac{w}{6} - \frac{w}{3l}(u''+p). \quad (108)$$

Substituting constants,

$$V = \frac{(230-u'')^2 - 25}{1380} + 19.167 - \frac{u''+10}{6},$$

whence we can form the following table of compression in the struts. The strains in the end braces from the constant-load equations, w being changed to $w'+w$.

Values of u''.	15	45	75	105
Strains in Tons.	45.00	34.78	22.39	11.30
Compression in	Aa & Yy	Dd & Vv	Gg & Ss	Kk & Pp

Adding the strains from the different simple truss, we have 165 tons for the total compression in Aa and Yy.

Multiplying V by 1.803 we have the tension in the ties as follows:

Values of u''.	15	45	75	105	135
Strains in Tons.	81.14	62.71	40.37	20.37	2.74
Tension in	Ad & Yv	Dg & Vs	Gk & Sp	Kn & Pm	Nq & Mi

The same equations for the horizontal strains, and for the constant-load vertical strains, will be obtained in any triple truss containing any odd number of panels.

97. A Simple Form of the Moving-Load Equation for either Simple Truss.—It will be noticed that the vertical equations for the moving loads in this and the previous case, Eqs. (80), (81), and (82), differ from the form $\frac{w'}{6l^2}(l-u)^2$, in the extreme case, only by the quantity $\frac{225}{1380}$; consequently, if this quantity be omitted, the difference in the result will be immaterial and upon the safe side, and we shall have but the simple form $\frac{w'}{6l^2}(l-u)^2$ to be added to the simple-truss constant-load equations. Moreover, all confusion will be avoided.

CASE VI. A QUADRUPLE TRUSS CONTAINING AN EVEN NUMBER OF PANELS.

98.—Let Fig. (38) represent a truss composed of four simple trusses, and containing an even number of panels.

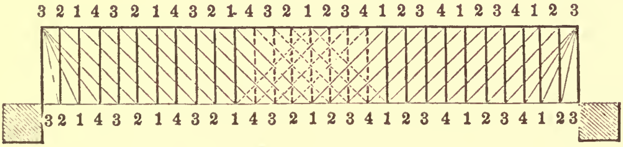


Fig. 38.

The simple trusses are numbered from the centre, as in the previous cases, and their different panel points are shown by the numbers in the figure.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel,
w = the weight,
x, x', x'', & x''' = distances from one abutment to the panel ends,
u, u', u'', & u''' = the distances to the centres of the panels of the simple trusses.
H & V = the horizontal and vertical strains.

99. Horizontal Strains.—Under the full uniform load, the horizontal strains are the greatest, and are determined as follows: the result would be the same if the quadruple truss contained any even number of panels.

In this figure,

Simple Truss No. 1 bears $\frac{w}{4}$,

" " 2 " $\frac{w}{4}+\frac{wp}{l}$,

" " 3 " $\frac{w}{4}$,

" " 4 " $\frac{w}{4}-\frac{wp}{l}$.

Taking moments around the panel points of Simple Truss No. 1, the moment of the abutment reaction is $\frac{wx}{8}$, the load on x is $\frac{w}{l}(x-2p)$, the distance of its centre of gravity from the point to which x is measured, $\frac{w}{2}+p$, whence we have

$$H=\frac{wx}{8d}-\frac{wx^2}{8dl}+\frac{wp^2}{2dl}, \quad \text{- - - - - - -} \quad (109)$$

for the horizontal strains in the upper and lower chords of this truss.

Taking moments around the panel points of Simple Truss No. 2, the moment of the abutment reaction is $\frac{wx}{8}+\frac{wpx}{2l}$, the load on x' is $\frac{w}{l}(x'-p)$, the distance of its centre of gravity from the point to which x' is measured is $\frac{x'}{2}+\frac{3p}{2}$, whence we have,

$$H'=\frac{wx'}{8d}-\frac{wx'^2}{8dl}+\frac{pwx'}{4dl}+\frac{3p^2w}{8dl}, \quad \text{- - -} \quad (110)$$

for the horizontal strains in the upper and lower chord of this truss.

Taking moments around the panel points of Simple Truss No. 3, the moment of the abutment reaction is $\frac{wx''}{8}$, and the moment of the load on x'' is, since this truss has a full panel end, $\frac{wx''^2}{8l}$, whence we have,

$$H'' = \frac{wx''}{8d} - \frac{wx''^2}{8dl}, \qquad (111)$$

for the horizontal strains in the upper and lower chords of this truss.

And taking moments around the panel points of Simple Truss No. 4, the moment of the abutment reaction is $\frac{wx'''}{8} - \frac{wpx'''}{8l}$, the load on x''' is $\frac{w}{l}(x''' - 3p)$, the distance of its centre of gravity from the point to which x''' is measured is $\frac{x''' + p}{2}$, whence we have,

$$H''' = \frac{wx'''}{8d} - \frac{wx'''^2}{8dl} - \frac{wpx'''}{4dl} + \frac{3p^2w}{8dl}. \qquad (112)$$

It is evident that the compression in the upper chord of the compound truss. at any point, x, is H at x, added to

$$\begin{array}{lll} H''' & \text{when } x''' & = x + p, \\ H'' & \text{"} \quad x'' & = x + 2p, \\ \text{and } H' & \text{"} \quad x' & = x + 3p, \end{array}$$

Similarly for the compression in the compound truss at the points x' x'' and x'''.

Substituting and adding, as before, we obtain,

$$H = \frac{w}{2d}\left(x + \frac{3p}{2}\right) - \frac{w}{2dl}\left(x + \frac{3p}{2}\right)^2 + \frac{9p^2w}{8dl}, \qquad (113)$$

for the compression at the panel points of Simple Trusses No. 1 and No. 4, and

$$H' = \frac{w}{2d}\left(x' + \frac{3p}{2}\right) - \frac{w}{2dl}\left(x + \frac{3p}{2}\right)^2 + \frac{p^2 w}{8dl}, \quad - \quad - \quad (114)$$

for the compression at the panel points of Simple Trusses No. 2 and No. 3.

Making $x = \frac{l}{2} - z$, and $x' = \frac{l}{2} - z'$, Eq. (113) becomes

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{3p}{2}\right)^2 + \frac{9p^2 w}{8dl}, \quad - \quad - \quad - \quad (115)$$

where z is the distance from the centre of the truss to the panel points of Simple Trusses No. 1 and No. 4; and Eq. (114) becomes

$$H' = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{3p}{2}\right)^2 + \frac{p^2 w}{8dl}, \quad - \quad - \quad - \quad - \quad (116)$$

where z' is the distance from the centre to the panel points of Simple Trusses No. 2 and No. 3.

In the lower chord of the compound truss the tension at any point, x, is the simple-truss strain at x, added to

$$\begin{array}{rll} H' & \text{when } x' & = x - p, \\ H'' & \quad \text{"} \quad x'' & = x - 2p, \\ \text{and } H''' & \quad \text{"} \quad x''' & = x - 3p, \end{array}$$

and similarly for the points x', x'', and x'''. Whence we obtain, changing x and x' to $\frac{l}{2} - z$ and $\frac{l}{2} - z'$.

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z + \frac{3p}{2}\right)^2 + \frac{9p^2w}{8dl}, \quad - \quad - \quad - \quad (117)$$

for the panel points of Simple Trusses No. 1 and No 2; and

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' + \frac{3p}{2}\right)^2 + \frac{p^2w}{8dl}, \quad - \quad - \quad - \quad (118)$$

for the panel points of Simple Trusses No. 3 and No. 4; z and z' being measured from the centre of the truss.

It will be seen from these equations, that in this case also the horizontal strain in the two chords is the same between the same inclined members. Hence, in practice, we shall not need the last two equations. The three end members of the lower chord are not subject to the action or the strains of all the four simple trusses, and consequently Eqs. (117 and 118) are not applicable to them. The first, or end member of the lower chord is subject to the strain of only one simple truss when x is zero, and consequently the strain is zero; the second member is subject to the strain of the first member, and the strain of that simple truss whose point is one panel length from the abutment, and the third member has the strain of the second member added to the strain of that simple truss whose panel point is two panel lengths from the abutment.

These strains can be most readily determined from the vertical equations for the constant load, and are given below.

100. Vertical Strains from a Constant Load.—In this case, as in the others, the simple trusses are entirely independent of each other in their vertical action under a uniform constant load; hence the equations are to be obtained from the simple-truss horizontal equations; whence, by the process previously described, we have, from Eqs. (109, 110, 111 and 112),

$$V = \frac{w}{8} - \frac{wu}{4l}, \quad - \; - \; - \; - \; - \; - \quad (119)$$

for Simple Truss No. 1,

$$V' = \frac{w}{8} - \frac{w}{4l}(u' - p), \quad - \; - \; - \; - \; - \quad (120)$$

for Simple Truss No. 2,

$$V'' = \frac{w}{8} - \frac{wu''}{4l}, \quad - \; - \; - \; - \; - \; - \; - \quad (121)$$

for Simple Truss No. 3, and

$$V''' = \frac{w}{8} - \frac{w}{4l}(u''' + p), \quad - \; - \; - \; - \quad (122)$$

for Simple Truss No. 4.

In which u, u', u'', and u''' are the distances from the abutment to the centres of the panels of the respective simple trusses.

101. Horizontal Tension in the Lower-Chord End Members.—The strains in these, referred to above, may be found as follows:

Let Fig. 39 represent the four end panels in a quadruple truss, and let d be a panel point in Simple Truss

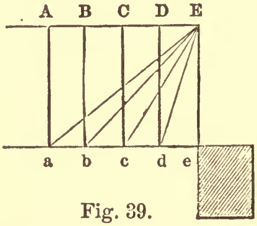


Fig. 39.

No. 4, and c a panel point in Simple Truss No. 3. The strain in bc is the strain in the latter truss at c added to that in the former at d.

The reaction of the abutment, or the vertical strain in Ee, from Simple Truss No. 4 is Eq. (122), where $u''' = -p$, $\therefore\ V = \frac{w}{8}$; and from moments around D,

$$H = \frac{wp}{8d}, \qquad - \quad - \quad - \qquad (123)$$

is the tension in the second member, where Simple Truss No. 4 has a panel point distant p from the abutment.

The vertical reaction of the abutment from Simple Truss No. 3 is Eq. (121), where $u'' = 0$, whence $V = \frac{w}{8}$; and from moments around C,

$$H = \frac{wp}{4d}.$$

This added to Eq. (120) gives

$$H = \frac{3pw}{8d}, \qquad - \quad - \quad - \qquad (124)$$

tension in bc, or third member of the lower chord, where

Simple Truss No. 4 has a panel point distant p from the abutment.

Next, let d be a panel point in Simple Truss No. 3, and c consequently a panel point in Simple Truss No. 2. The reaction of the abutment from Simple Truss No. 3 is Eq. (121), where $u'' = -p$, whence $V = \frac{w}{8} + \frac{wp}{4l}$, and

$$H = \frac{pw}{8d} + \frac{p^2w}{4dl} \quad - \quad - \quad (125)$$

strain in the second member; and the reaction of the abutment from Simple Truss No. 2 is Eq. (120), where $u' = 0$, whence $V = \frac{w}{8} + \frac{wp}{4l}$, and $H = \frac{pw}{4d} + \frac{p^2w}{2dl}$, which added to Eq. (125) gives

$$H = \frac{3pw}{8d} - \frac{3p^2w}{4dl}, \quad - \quad - \quad (126)$$

for the tension in the third member of the lower chord where Simple Truss No. 3 has a panel point distant p from the abutment.

Next, let d be a panel point of Simple Truss No. 2, and c consequently a panel point of Simple Truss No. 1. The reaction of the abutment from the former is Eq. (120), where $u' = -p$, whence $V = \frac{w}{8} + \frac{pw}{2l}$, therefore,

$$H = \frac{pw}{8d} + \frac{p^2w}{2dl}, \quad - \quad - \quad - \quad (127)$$

is the tension in the second member; and the reaction from Simple Truss No. 1 is Eq. (119), where $u = 0$, whence $V = \frac{w}{8}$ and $H = \frac{pw}{4d}$, which, added to Eq. (127), gives

$$H = \frac{3pw}{8d} + \frac{p^2w}{2dl}, \qquad (128)$$

for the tension in the third member when Simple Truss No. 2 has a panel point distant p from the abutment.

The remaining case is where d is a panel point of Simple Truss No. 1, and c of Simple Truss No. 4. The reaction of the abutment from the former is Eq. (119), where $u = -p$, whence $V = \frac{w}{8} + \frac{pw}{4l}$, and

$$H = \frac{pw}{8d} + \frac{p^2w}{4dl}, \qquad (129)$$

is the tension in the second member; and the reaction of the abutment from Simple Truss No. 4 is Eq. (122), where $u''' = 0$, whence $V = \frac{w}{8} - \frac{pw}{4l}$, and $H = \frac{pw}{4d} - \frac{p^2w}{2dl}$, which, added to Eq. (129), gives

$$H = \frac{3pw}{8d} - \frac{p^2w}{4dl}, \qquad (130)$$

for the tension in the third member of the lower chord when Simple Truss No. 1 has a panel point distant p from the abutment.

102. Vertical Strains from a Moving Load.—Let Figs. 40, 41, 42, and 43 represent the different ends of the four simple trusses. Let the vertical lines show the divisions into simple-truss panels, and the points in the lower chord the panel points of the compound truss.

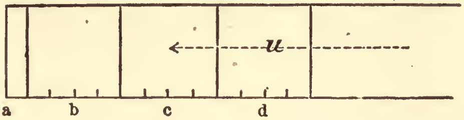


Fig. 40.

In Fig. 40 u is the distance from the right abutment to the centres of the simple-truss panels or the points a, b, c, d, &c.; w' being the weight of the full movable load, $\frac{1}{4}$ of $\frac{w'}{l}(l-u+p)$ is the load on the panel points of this truss within the distance $l-u$; and the distance of its centre of gravity from the left abutment is $\frac{l-u-p}{2}$, whence the reaction of the other abutment, or the greatest vertical strain from the movable load upon the simple truss whose loaded end has a panel $= p$, is,

$$V = \frac{w'}{8l^2}[(l-u)^2 - p^2]. \quad - \quad - \quad (131)$$

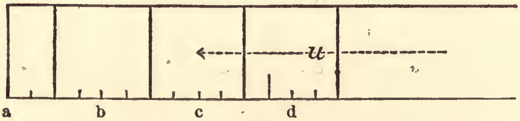


Fig. 41.

In Fig. 41 u' is the distance from the right abutment to the centre of the simple-truss panels, or the points a, b, c, d, &c.; whence $\frac{w}{4l^2}(l-u')^2$ is the load on

$l-u'$, distance of its centre of gravity from the loaded end, $\frac{l-u'}{2}$, and the greatest vertical strain upon the simple truss whose panel at the loaded end $= 2p$, is

$$V = \frac{w'}{8l^2}(l-u')^2. \quad - \quad - \quad - \quad (132)$$

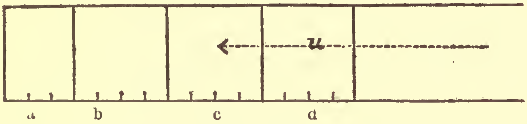


Fig. 42.

In Fig. 42, similarly, the load on $l-u''$ is $\frac{w'}{4l}(l-u''-p)$, the distance of its centre of gravity from the loaded end, $\frac{l-u''+p}{2}$, whence the greatest vertical strain upon the simple truss whose panel at the loaded end $= 3p$, is

$$V = \frac{w'}{8l^2}[(l-u'')^2-p^2] \quad \cdot \quad (133)$$

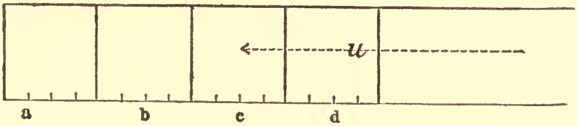


Fig. 43.

In Figure 43, similarly, the load on $l-u'''$ is $\frac{w'}{4l}(l-u'''-2p)$, the distance of its centre of gravity from the loaded end, $\frac{l-u'''+2p}{2}$, whence the greatest vertical strain upon the simple truss whose panel end at the loaded end $= 4p$ is

$$V = \frac{w'}{8l^2}[(l - u''')^2 - 4p^2]. \quad - \quad - \quad (134)$$

In this case, Simple Trusses No. 1 and No. 3 are continuous, and their panels uniform throughout, but one half of Simple Truss No. 2 is united to the opposite half of Simple Truss No. 4.

It has been fully explained how to determine which moving-load vertical equation is to be added to the constant-load vertical equation belonging to any simple truss.

In the truss shown in Fig. 38, Eq. (131) is to be added to Eq. (122), Eq. (132) to Eq. (119), Eq. (133) to Eq. (120), and Eq. (134) to Eq. (121.)

103. Example.—Let Fig. 44 represent a quadruple truss containing an even number of panels, and loaded upon the lower chord.

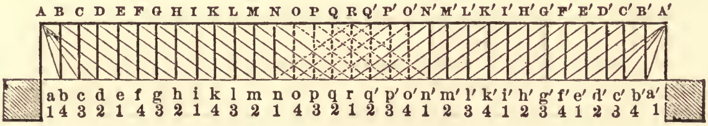


Fig. 44.

Let l = 320 feet, the length of the truss,
d = 27.5 the depth of the truss,
p = 10 feet, the length of a panel,
w' = 208 tons, the weight of the full uniform movable load,
w = 144 tons, the weight of the truss.

Values of z.	Values of z'.	Strains in Tons.	Compression in	Tension in
0		512	QR and RQ'	
	10	512	PQ and Q'P'	
	20	512	OP and P'O'	
30		512	NO and O'N'	
40		504	MN and N'M'	qr and r'q'
	50	488	LM and M'L'	pq and q'p'
	60	472	KL and L'K'	op and p'o'
70		456	IK and K'I'	no and o'n'
80		432	HI and I'H'	mn and n'm'
	90	400	GH and H'G'	lm and m'l'
	100	368	FG and G'F'	kl and l'k'
110		336	EF and F'E'	ik and k'i'
120		296	DE and E'D'	hi and i'h'
	130	248	CD and D'C'	gh and h'g'
	140	200	BC and C'B'	fg and g'f'
150		152	AB & B'A'	ef and f'e'
130		96		de and e'd'
		48		cd and d'c'
		16		bc and c'b'

104. Horizontal Strains.—Substituting these values in Eqs. (115) and (116) for the upper-chord strains, in Eq. (117) for the fourth lower-chord member, and in Eqs. (123) and (124) for the second and third lower-chord members, since Simple Truss No. 4 has a panel point distant p from the abutment; the other lower-chord strains being taken from the upper-chord strains between the same inclined braces; w in these equations being $w' + w$, or 352 tons; we have the preceding table; there is no strain in ab and $b'a'$.

105. Vertical Strains.—Since Simple Truss No. 1 has a panel end $= 4p$, Eq. (134) is to be added to Eq. (119) to obtain the maximum vertical strains in this truss; Simple Truss No. 2 has a panel end $= 3p$, therefore Eq. (133) is to be added to Eq. (122) to obtain the greatest vertical strains in No. 4; No. 3 has a panel end $= 2p$, therefore Eq. (132) is to be added to Eq. (121); and No. 4 has a panel end $= p$, therefore Eq. (131) is to be added to Eq. (120) to obtain the maximum vertical strain in Simple Truss No. 2.

Making the variable, in the first case, u, in the second, u', in the third, u'', and in the fourth, u''', substituting the values above, w of the constant-load equations being 144 tons, and multiplying the vertical strain by secants of the tie angles, we can form the following table of strains in the braces. The secants are 1.765, 1.48, 1.236, and 1.063.

Total compression on Aa and A′a′ is 170.5 tons.

The strains in the end braces of those simple trusses which are uniform beyond the centre, No. 1 and No. 3, are found from the same equations which give the strains in the other braces; but in the end braces of No. 2 and No. 4 the strains are determined from the constant-load portion of the equations, $w' + w$ being put for w.

Values of u.	Values. of u'	Values of $u.''$	Values of u'''	Strains in Tons.	Compression in	Strains in Tons.	Tension in
	—10			44.00	Aa & A'a'	46.77	Ab & A'b'
		0		44.00	Aa & A'a'	54.38	Ac & A'c'
			10	44.00	Aa & A'a'	65.12	Ad & A'd'
20				38.50	Aa & A'a'	67.95	Ae & A'e'
	30			34.83	Bb & B'b'	61.47	Bf & B'f'
		40		33.41	Cc & C'c'	58.97	Cg & C'g'
			50	31.98	Dd & D'd'	56.53	Dh & D'h'
60				28.31	Ee & E'e'	49.97	Ei & E'i'
	70			24.84	Ff & F'f'	43.84	Fk & F'k'
		80		23.62	Gg & G'g'	41.69	Gl & G'l'
			90	22.41	Hh & H'h'	39.55	Hm & H'm'
100				18.94	Ii & I'i'	33.43	In & I'n'
	110			15.67	Kk & K'k'	27.66	Ko & K'o'
		120		14.66	Ll & L'l'	25.87	Lp & L'p'
			130	13.64	Mm & M'm'	24.07	Mq & M'q'
140				10.36	Nn & N'n'	18.29	Nr & N'r'
	150			7.31	Oo & O'o'	12.90	Oq' & O'q
		160		6.50	Pp & P'p'	11.47	Pp' & P'p
			170	5.71	Qq & Q'q'	10.01	Qo' & Q'o
180				2.62	Rr'	4.62	Rn' & R'n

CASE VII.—A QUADRUPLE TRUSS, CONTAINING AN ODD NUMBER OF PANELS.

106.—In this quadruple truss, as in the others, the equations are not affected by the number of panels the truss contains, but by the condition whether the number be odd or even. Hence, it is necessary to determine equations for this case. It should be remembered that, in determining equations for any case, the simplest form of truss may be taken, provided it fulfils the necessary conditions.

Let Fig. 45 represent a quadruple truss containing an odd number of panels. This, like the other, is divisible into four simple trusses, numbered, as in the

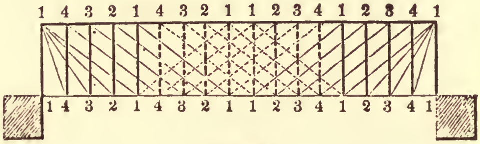


Fig. 45.

previous cases, from the centre towards the abutments, and whose panel points are shown by the numbers.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel of the compound truss,
w = the maximum weight uniformly distributed,
$x, x' x'', \& x'''$ = the distances from one abutment to the panel points of the simple trusses,
$u, u', u'' \& u'''$ = the distances from the same abutment to the centres of the simple-truss panels,
H & V = the horizontal and vertical strains.

107. Horizontal Strains.—The moment of the load on a segment of any simple truss in this case is the same as the moment of the load on the same segment of that simple truss in the previous case whose end panel is similar.

In this figure,

Simple Truss No. 1 bears $\frac{1}{4}\left(w - \frac{wp}{l}\right) + \frac{wp}{l}$,

" " 2 " $\frac{1}{4}\left(w - \frac{wp}{l}\right)$,

" " 3 " $\frac{1}{4}\left(w - \frac{wp}{l}\right)$,

" " 4 " $\frac{1}{4}\left(w - \frac{wp}{l}\right)$,

whence,

$$H' = \frac{wx}{8d} - \frac{wx^2}{8dl} + \frac{3pwx}{8dl}, \quad - \quad - \quad - \quad (135)$$

for Simple Truss No. 1;

$$H = \frac{wx'}{8d} - \frac{wx'^2}{8dl} + \frac{pwx'}{8dl} + \frac{3p^2w}{8dl}, \quad - \quad - \quad (136)$$

for Simple Truss No. 2;

$$H' = \frac{wx''}{8d} - \frac{wx''^2}{8dl} - \frac{pwx''}{8dl} + \frac{p^2w}{2dl} \quad - \quad - \quad (137)$$

for Simple Truss No. 3; and,

$$H''' = \frac{wx'''}{8d} - \frac{wx'''^2}{8dl} - \frac{3pwx'''}{8dl} + \frac{3p^2w}{8dl}, \quad - \quad (138)$$

for Simple Truss No. 4.

These are the horizontal strains in the upper and lower chords of the simple trusses. Adding them in the same manner as in the previous cases we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{3p}{2}\right)^2 + \frac{3p^2w}{8dl}, \quad - \quad - \quad (139)$$

z being the distance from the centre of the truss, for the compressions in the upper chord at the panel points of Simple Trusses No. 1 and No. 3.

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{3p}{2}\right)^2 - \frac{p^2w}{8dl}, \quad - \quad - \quad (140)$$

z' being the distance from the centre of the truss, for the compressions in the upper chord at the panel points of Simple Truss No. 2, and,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z'' - \frac{3p}{2}\right)^2 + \frac{15p^2w}{8dl}, \quad - \quad - (141)$$

z'' being the distance from the centre of the truss, for the compressions in the upper chord at the panel points of Simple Truss No. 4.

For the tension in the lower chord we obtain,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z + \frac{3p}{2}\right)^2 + \frac{15p^2w}{8dl}, \quad - \quad - (142)$$

at the panel points of No. 1;

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' + \frac{3p}{2}\right)^2 + \frac{3p^2w}{8dl}, \quad - \quad - \quad (143)$$

at the panel points of No. 2 and No. 4; and

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z'' + \frac{3p}{2}\right)^2 - \frac{p^2w}{8dl}, \quad - \quad - \quad - (144)$$

at the panel points of No. 3.

108. Vertical Strains from a Constant Load.—From Eqs. (135, 136, 137, and 138) we have,

$$V = \frac{w}{8} - \frac{w}{4l}\left(u - \frac{3p}{2}\right), \quad - \quad - \quad (145)$$

for No. 1;

$$V = \frac{w}{8} - \frac{w}{4l}\left(u' - \frac{p}{2}\right), \quad - \quad - \quad - \quad (146)$$

for No. 2;

$$V = \frac{w}{8} - \frac{w}{4l}\left(u'' + \frac{p}{2}\right), \quad - \quad - \quad - \quad (147)$$

for No. 3; and

$$V = \frac{w}{8} - \frac{w}{4l}\left(u''' + \frac{3p}{2}\right), \quad - \quad - \quad - \quad (148)$$

for No. 4.

109. Horizontal Tensions in Lower-Chord End Members.—The strains are obtained as in (99). See Fig. 39, whence,

$$H = \frac{pw}{8d} - \frac{p^2w}{8dl}, \quad - \quad - \quad - \quad (149)$$

in the second member; and

$$H = \frac{3pw}{8d} - \frac{3p^2w}{8dl}, \quad - \quad - \quad - \quad (150)$$

in the third member, when a panel point of No. 4 is distant p from the abutment.

$$H = \frac{pw}{8d} + \frac{p^2w}{8dl}, \quad - \quad - \quad - \quad - \quad (151)$$

in the second member; and

$$H = \frac{3pw}{8d} + \frac{3p^2w}{8dl}, \quad - \quad - \quad - \quad (152)$$

in the third member, when a panel point of No. 3 is distant p from the abutment.

$$H = \frac{pw}{8d} + \frac{3p^2w}{8dl}, \quad - \quad - \quad - \quad - \quad (153)$$

in the second member, and

$$H = \frac{3pw}{8d} + \frac{9p^2w}{8dl}, \quad - \quad - \quad - \quad - \quad (154)$$

in the third member, when a panel point of No. 2 is distant p from the abutment. And

$$H = \frac{pw}{8d} + \frac{5p^2w}{8dl}, \quad - \quad - \quad - \quad - \qquad (155)$$

in the second member; and

$$H = \frac{3pw}{8d} - \frac{p^2w}{8dl}, \quad - \quad - \quad - \quad - \qquad (156)$$

in the third member, when a panel point of No. 1 is distant p from the abutment.

110. Vertical Strains from a Moving Load.—Eqs. (131), (132), (133) and (134) apply in this case. None of the simple trusses are continuous, but one half of No. 1 is connected by the counterbracing with the opposite half of No. 4, and one half of No. 2 is similarly connected with the opposite half of No. 3; hence care must be exercised in the addition of the vertical equation, and it is always safest to make a diagram of the simple trusses, as connected under the moving load.

111. Example.—Let Fig. 46 represent a quadruple truss containing an odd number of panels.

Let l = 328 feet, the length of the truss,
d = 32 feet, the depth of the truss,
p = 8 feet, the length of the panel,
w' = 328 tons, the weight of the full uniform movable load,
w = 164 tons, the constant truss load.

The load is upon the lower chord.

Values of z.	Values of z'.	Values of z''.	Strains in Tons.	Compression in	Tension in
4			630	UV VV′ & V′U′	
	12		630	TU & U′T′	
20			630	ST & S′T′	
		28	630	RS & S′R′	vv′
36			618	QR & R′Q′	uv & v′u′
	44		606	PQ & Q′P′	tu & u′t′
52			594	OP & P′O′	st & t′s′
		60	582	NO & O′N′	rs & s′r′
68			558	MN & N′M′	qr & r′q′
	76		534	LM & M′L′	pq & q′p′
84			510	KL & L′K′	op & p′o′
		92	486	IK & K′I′	no & o′n′
100			450	HI & I′H′	mn & n′m′
	108		414	GH & H′G′	lm & m′l′
116			378	FG & G′F′	kl & l′k′
		124	342	EF & F′E′	ik & k′i′
132			294	DE & E′D′	hi & i′h′
	140		246	CD & D′C′	gh & h′g′
148			198	BC & C′B′	fg & g′f′
		156	150	AB & B′A′	ef & f′e′
	140		90		de & e′d′
			45		cd & d′c′
			15		bc & c′b′

112. Horizontal Strains.—For the upper chord, use Eqs. (139), (140) and (141); for the lower chord, take the strains from the upper chord between the same inclined braces, and beyond these, for the fourth member, use Eq. (143), for the third member Eq. (150), and for the second, Eq. (149); w of these equations being made equal to $w' + w$, or 492 tons. Hence we can form the preceding table of strains in the chords.

113. Vertical Strains.—For the strains in the struts of Simple Truss No. 1, add Eq. (131) to Eq. (145). In No. 2 add Eq. (132) to Eq. (146). In No. 3 add Eq. (133) to Eq. (147), and in No. 4 add Eq. (134) to Eq. (148). For the ties multiply the vertical strain so obtained by the secants of the tie angles, which are 1.414, 1.25, 1.118 and 1.0308. The following gives the strains in the braces:

Fig. 46.

Values of u.	Values of u'.	Values of u''.	Values of u'''.	Strains in Tons.	Compression in	Strains in Tons.	Tension in
			—8	60.00	Aa & A'a'	63.24	Ab & A'b'
		0		60.00	Aa & A'a'	72.11	Ac & A'c'
	8			60.00	Aa & A'a'	84.84	Ad & A'd'
16				60.00	Aa & A'a'	100.00	Ae & A'e'
			24	51.12	Bb & B'b'	72.28	Bf & B'f'
		32		49.36	Cc & C'c'	69.80	Cg & C'g'
	40			47.61	Dd & D'd'	67.32	Dh & D'h'
48				45.85	Ee & E'e'	64.83	Ei & E'i'
			56	40.10	Ff & F'f'	56.70	Fk & F'k'
		64		38.53	Gg & G'g'	54.48	Gl & G'l'
	72			36.97	Hh & H'h'	52.28	Hm & H'm'
80				35.41	Ii & I'i'	49.69	In & I'n'
			88	29.85	Kk & K'k'	42.21	Ko & K'o'
		96		28.49	Ll & L'l'	40.28	Lp & L'p'
	104			27.12	Mm & M'm'	38.35	Mq & M'q'
112				25.76	Nn & N'n'	36.42	Nr & N'r'
			120	20.39	Oo & O'o'	28.83	Os & O's'
		128		19.22	Pp & P'p'	27.18	Pt & P't'
	136			18.05	Qq & Q'q'	25.52	Qu & Q'u'
144				16.88	Rr & R'r'	23.87	Rv & R'v'
			152	11.71	Ss & S's'	16.71	Sv' & S'v
		160		10.73	Tt & T't'	15.17	Tu' & T'u
	168			9.75	Uu & U'u'	13.79	Ut' & U't
176				8.78	Vv & V'v'	12.41	Vs' & V's
			184			5.37	V'r' & Vr
		192				4.27	U'q' & Uq
	200					3.17	T'p' & Tp
208						2.06	S'o' & So

Total compression in Aa and A′a′ is 240 tons; the strains in the end braces determined as in the previous cases.

114. Other Compound Trusses.—In the same manner equations may be deduced for any compound truss with vertical struts and inclined ties composed of any number of simple trusses, and so many examples have been given that the process can present no difficulties.

115. Simple Forms of Equations.—The difference between the other moving-load equations in the last case and $\frac{w'}{8l^2}(l-u)^2$ is very slight, and being always on the safe side, the latter form may be used in practice for all the simple trusses in this case, and all difficulty in the addition of equations thereby avoided.

CHAPTER V.

TRUSSES WITH HORIZONTAL CHORDS, WITH STRUTS AND TIES OF EQUAL INCLINATIONS, SUBJECT TO CONSTANT AND TO MOVING LOADS.

CASE I.—A SIMPLE TRUSS.

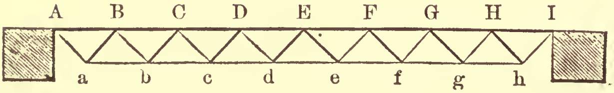


Fig. 47.

116. Horizontal Strains.—What is generally termed a Warren Girder, shown in Fig. 47, is an example of this description of truss, supporting the load upon the upper chord. In this case moments may be taken around the panel points of the upper chord to obtain the tension in the members of the lower chord vertically beneath these points, and since the load may be considered as concentrated at these points, we have the same conditions which in (28) gave us Eq. (14),

$$H = \frac{wx}{2d} - \frac{wx^2}{2dl},$$

for the maximum horizontal strain. This equation can be applied only to the lower chord. For the strain in the upper chord, moments must be taken around the panel points in the lower chord, since it is only at these points that a vertical section can be made which does not cut more than one member (having a moment) subject to a horizontal strain.

Let x' be the horizontal distance from one abutment to any one of the lower-chord panel points, w the full uniform load, and p the distance between the points; and we can form the equation,

$$H' = \frac{wx'}{2d} - \frac{wx'^2}{2dl} - \frac{wp^2}{8dl}, \quad \cdot \quad \cdot \quad (157)$$

for the compressions in the members of the upper chord, opposite the lower-chord panel points, distant x' from the abutment.

117. Vertical Strains from a Constant Load.—Eq. (14), which gives the lower-chord tension, gives also the compression in the upper chord at the point to which x is measured; that is, the resultant compression from the strains in the braces and the chord-member on one side of the point. Similarly, Eq. (157) gives the tension at the points in the lower chord. Therefore, if the first be the strain at F, as it is at the same time in ef, and the second the strain at f, as it is at the same time in FG, the difference between the two will be the horizontal component of the strain in the brace Ff, and its vertical component may be obtained from the proportion of $x - x' : d$.

Performing this operation, we have for the difference between Eqs. (14) and (157), or the horizontal component of the strain in the brace,

$$H - H' = \frac{w}{2d}(x - x') - \frac{w}{dl}(x - x')\left(\frac{x + x'}{2}\right) + \frac{wp^2}{8d}.$$

Since $p = 2(x - x')$, we have from the proportion above,

$$V = \frac{w}{2} - \frac{w}{l}\left(\frac{x + x'}{2} - \frac{p}{4}\right); \quad - \quad - \quad (158)$$

and as

$$x = x' + \frac{p}{2},$$

$$V = \frac{w}{2} - \frac{wx'}{l} \quad - \quad - \quad - \quad (159)$$

is the vertical component of the strain in Ff.

If Eq. (14) next represent the strain at G, and be subtracted from Eq. (157), representing the strain at f, we shall again obtain

$$V = \frac{w}{2} - \frac{wx'}{l},$$

for the vertical component of the strain in fG. This equation is the same as Eq. (18), x' here being the same as u, that is the distance from the abutment to midway between the loaded points.

118. Vertical Strains from the Moving Load.—As this case comprises but a single system, one panel point cannot be fully loaded without a portion of the weight coming upon the next point, and thus rendering the conditions similar to those of the case in (54), and consequently Eq. (37) applies when the truss is half or more than half loaded, and Eq. (39) when less than half loaded.

119.—Example.—In Fig. 47,

Let $l = 80$ feet, the length of the truss,
$d = 5$ feet, the depth of the truss,
$p = 10$ feet, the distance between the panel points, in the same chord.
$w' = 80$ tons, the weight of the full uniform movable load,
$w = 40$ tons, the weight of the constant load.

Substituting the values of these constants in Eqs. (14) and (157), we can form the following table of horizontal strains :

Values of x.	10	20	30	40				
Values of x'.					5	15	25	35
Strains in Tons.	105	180	225	240	52.5	142.5	202.5	232.5
Compression in					AB & HI	BC & GH	CD & FG	DE & EF
Tension in	ab & gh	bc & fg	cd & ef	de				

For the maximum vertical strain we have Eq. (37), see (54),

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'\left(l - u - \frac{p}{2}\right)^2}{2l(l - p)},$$

when the load covers half and more than half the truss; and Eq. (39),

$$V = \frac{w}{2} - \frac{wu}{l} + \frac{w'\left(l - u - \frac{p}{2}\right)^2}{2(l-p)^2},$$

when the load covers less than half the truss.

In these equations u is the distance from the abutment to a point midway between two panel points in the loaded chord, and the equations give the vertical components of the strains in the braces between these two points. As long as V has a plus value, that brace whose loaded chord end is nearer to the abutment from which u is measured is a tie, while that brace whose unloaded chord end is nearer is a strut.

As the equations are applied to the different ends of the truss, it will be seen that some of the braces near the centre act sometimes as struts and sometimes as ties; these are termed counterbraces. Substituting the values of the constants, and multiplying by 1.414 the secant of the angle of all the braces,

Values of u.	5	15	25	35	45
Strains in Tons.	74.24	54.03	35.86	19.70	6.85
Compression in	Ba & Hh	Cb & Gg	Dc & Ff	Ed & Ee	Fe & Dd
Tension in	Aa & Ih	Bb & Hg	Cc & Gf	Dd & Fe	Ee & Ed.

CASE II.—A DOUBLE TRUSS CONTAINING AN EVEN NUMBER OF PANELS.

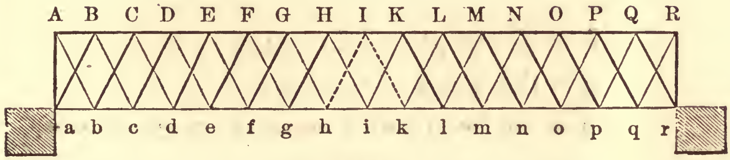


Fig. 48.

120.—Let Fig. 48 represent a truss composed of the two simple trusses of Figs. (49) and (50). The first of which will be termed Simple Truss No. 1, since its braces meet at the centre, and the other Simple Truss No. 2.

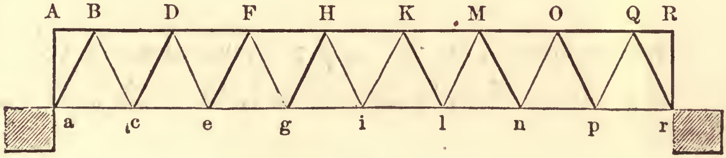


Fig. 49.

In these figures those braces which act solely as counterbraces are removed.

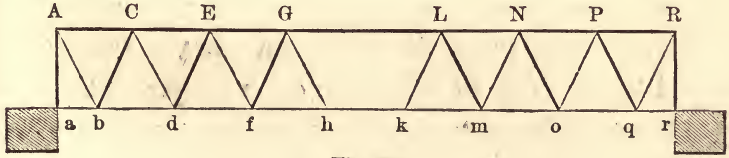


Fig. 50.

In these trusses the vertical strains, or those strains having vertical components, are entirely independent of each other; and, as in the previous cases of compound trusses, the horizontal strain in the compound truss is the sum of the strains in the simple trusses.

121. Horizontal Strains.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel or chord-member.
w = the full weight, uniformly distributed.
x, x' &c. = the distance from the abutment to any panel point.

Each simple truss bears half the load which is upon the lower chord.

$$\therefore \qquad H = \frac{wx}{4d} - \frac{wx^2}{4dl}, \qquad (160)$$

is the compression in the upper chord opposite to the loaded points of Simple Truss No. 1, to which x is the distance, and

$$H' = \frac{wx'}{4d} - \frac{wx'^2}{4dl} + \frac{wp^2}{4dl}, \qquad (161)$$

is the compression in the upper chord opposite to the loaded points of Simple Truss No. 2, to which x' is the distance. The value of H in one simple truss at x, added to the value of H′ in the other simple truss at a point nearer the centre by the value of p, will give the amount of horizontal strain in the upper chord of the double truss between x and $x+p$, or in that member of the upper chord which is on the centre side of the point to which x is measured.

Making x' of Eq. (161) equal to $x + p$ of Eq. (160), and adding the two equations, we have,

$$H = \frac{w}{2d}\left(x + \frac{p}{2}\right) - \frac{w}{2dl}\left(x + \frac{p}{2}\right) + \frac{wp^2}{8dl}, \qquad (162)$$

and making $x = \frac{l}{2} - z$, we have,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 + \frac{wp^2}{8dl}. \quad - \quad (163)$$

Here z is the distance from the centre of the truss to the abutment end, and $z - \frac{p}{2}$ consequently the distance to the centre end of that upper-chord member whose strain is given by H.

If x of Eq. (160) be made equal to $x' + p$ of Eq. (161), and the two equations added, we shall have the same result, or Eq. (163) will give the compression in all the members of the upper chord.

In Fig. (49),

$$H = \frac{wx}{4d} - \frac{wx^2}{2dl} - \frac{wp^2}{4dl}, \quad - \quad - \quad (164)$$

is the tension in the lower chord opposite the upper-chord panel points to which x is measured.

In Fig. 50,

$$H' = \frac{wx'}{4d} - \frac{wx'^2}{4dl}, \quad - \quad - \quad - \quad (165)$$

is the tension in the lower chord opposite the upper-chord panel points to which x' is measured.

The tension in this case, in one simple truss at x, added to the tension in the other simple truss, when $x' = x + p$, will give the tension in that member of the double truss on the centre side of the point to which x is measured.

Making x' of Eq. (165) equal to $x + p$, and adding to Eq. (164), or making x of Eq. (164) equal to $x' + p$, and adding to Eq. (165), we obtain,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 - \frac{3wp^2}{8dl}, \quad - \quad - \quad - \quad (166)$$

for the tension in any member of the lower chord, where $z - \frac{p}{2}$ is the distance from the centre of the truss to the centre of the member.

122. Vertical Strains from a Constant Load.—It will be seen from the last case that the constant-load vertical equations may be obtained from the simple-truss horizontal equations, as in the previous cases; that is, from the difference in these horizontal equations applying to the same chord. In the last case the vertical equation was obtained from the difference in the horizontal strains at the two ends of the same brace; but it would be the same had it been obtained from the difference in the upper-chord equation alone, at two ends of the same panel. The proportion was $\frac{p}{2} : d$; but if we take the horizontal strains at the two ends of a panel of the simple truss, their difference is just double the difference at the two ends of a brace, because, between two weights or loaded panel points, the horizontal strains or horizontal components of the strains in the two braces are equal; the proportion will therefore be $p : d$, whence we have for either simple truss,

$$V = \frac{w}{4} - \frac{wu}{2l}, \qquad (167)$$

in which u is the distance from the abutment to the centre of any loaded chord-member of the simple trusses.

123. Vertical Strains from a Moving Load.—Under a moving load the simple trusses present the same cases as Fig. 21 and Fig. 22; and Eq. (46) applies to that simple truss having a full end panel, or in this case to Fig. 49; and Eq. 45 to that simple truss having an end panel of half the length of its other panels, in this case to Fig. 50. Each of these is to be added to constant-load equation (167).

124. Example.—Let Fig. 48 represent a truss in which
l = 160 feet, the length of the truss,
d = 20 feet, the depth of the truss,
p = 10 feet, the length of a panel or chord-member,
w' = 160 tons, the weight of a full movable load,
w = 80 tons, the constant-truss weight.

The load is upon the lower chord.

Substituting the values given above in Eqs. (163) and (166), we form the following table of strains in the upper and lower chords:

Values of $z - \frac{p}{2}$.	5	15	25	35	45	55	65	75
Strains in Tons.	240	232.5	217.5	195	165	127.5	82.5	30
Compression in	HI & IK	GH & KL	FG & LM	EF & MN	DE & NO	CD & OP	BC & PQ	AB & QR
Strains in Tons.	236.25	228.75	213.75	191.25	161.25	123.75	78.70	26.25
Tension in	hi & ik	gh & kl	fg & lm	ef & mn	de & no	cd & op	bc & pq	ab & qr

For the vertical strains in Simple Truss No. 1, we add Eq. (46) to Eq. (167), and multiply by 1.118, the secant of the brace angle; for strains in No. 2, add Eq. (45) to Eq. (167) and multiply by 1.118; whence we can form the following table of strains in the braces:

Values of u in Truss No. 1.	Values of u in Truss No. 2.	Strains in Tons.	Compression in	Tension in
	0	67.08		Ab & Rq
10		58.69	Ba & Qr	Bc & Qp
	20	52.43	Cb & Pq	Cd & Po
30		43.32	Dc & Op	De & On
	40	37.90	Ed & No	Ef & Nm
50		29.35	Fe & Mn	Eg & Ml
	60	24.60	Gf & Lm	Gh & Lk
70		15.55	Hg & Kl	Hi & Ki
	80	12.75	Ih & Ik	Ik & Ih
90		5.59	Ki & Hi	Kl & Hg

Total compression in Aa and Rr, 112.5 tons.

The braces which act as counterbraces, or which are subject to both tension and compression, are Ih, Ik, Hi, Kl, Hg, and Ki.

CASE III.—A DOUBLE TRUSS CONTAINING AN ODD NUMBER OF PANELS.

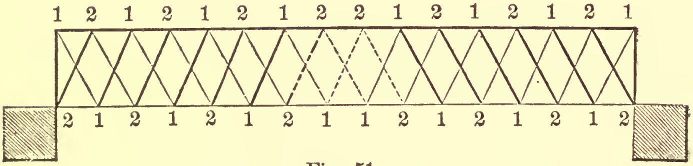


Fig. 51.

125. Horizontal Strains.—This truss is composed, like the previous one, of two simple trusses, the panel points of which are numbered in the figure in the same manner as before, and the braces which act solely as counterbraces shown by the dotted lines.

Proceeding in the same manner as before, we obtain for the horizontal strains in the upper chord,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z + \frac{p}{2}\right)^2 + \frac{3w^2p}{8dl}, \quad - \; - \; (168)$$

where z is the distance from the centre to the panel points of Simple Truss No. 1 in the upper chord, and H is the strain in that member on the abutment side of the point to which z is measured.

$$H' = \frac{wl}{8d} - \frac{w}{2dl}\left(z' + \frac{p}{2}\right)^2 - \frac{wp^2}{8dl}, \quad - \; - \; (169)$$

where z' is the distance from the centre to the panel points of Simple Truss No. 2 in the upper chord, and H′ is the strain in that member on the abutment side of the point to which z' is measured.

In the lower chord,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 - \frac{wp^2}{8dl}, \quad - \quad - \quad (170)$$

z being the distance to the panel points of No. 1 in the lower chord, and H the tension in the member on the centre side of the point to which z is measured; and

$$H' = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{p}{2}\right) - \frac{5wp^2}{8dl}, \quad - \quad - \quad (171)$$

z being the distance to the panel points of No. 2, and H′ the tension in the member on the centre side of the point to which z' is measured. In all these equations w is the maximum uniform load, or equal to $w' + w$ of the examples previously given.

126. Vertical Strains from a Constant Load.—The vertical equation for a constant load upon No. 1 is,

$$V = \frac{w}{4} - \frac{w}{2l}\left(u - \frac{p}{2}\right), \quad - \quad - \quad - \quad (172)$$

and for the constant load upon No. 2,

$$V = \frac{w}{4} - \frac{w}{2l}\left(u' - \frac{p}{2}\right). \quad - \quad - \quad - \quad (173)$$

127. Vertical Strains from a Moving Load.—The vertical equations for the moving load are the same as in the last case, and their application is governed by the same principles which have been explained in the previous examples of double trusses. An example of the application of the equations belonging to this case is unnecessary.

CASE IV.—A QUADRUPLE TRUSS CONTAINING AN EVEN NUMBER OF PANELS.

Fig. 52.

128.—Omitting the braces which are used solely as counterbraces, and numbering as before, we have the four simple trusses shown in Fig. (53).

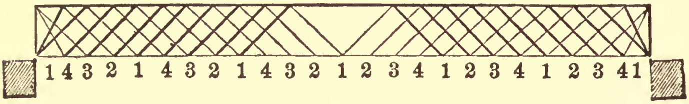


Fig. 53.

129. Horizontal Strains.—In this figure the weight borne by each truss is $\frac{w}{4}$, the load being upon the lower chord.

By reasoning as in the previous case of a quadruple truss, we obtain, for the upper chord of Simple Truss No. 1,

$$H = \frac{wx}{8d} - \frac{wx^2}{8dl}, \quad - \quad - \quad - \quad (174)$$

x being the distance to a point in the lower chord.

For No. 2,

$$H' = \frac{wx'}{8d} - \frac{wx'^2}{8dl} + \frac{pwx'}{4dl} + \frac{3p^2w}{8dl}. \quad - \quad - \quad (175)$$

For No. 3,

$$H'' = \frac{wx''}{8d} - \frac{wx''^2}{8dl} + \frac{p^2w}{2dl}. \quad - \quad - \quad - (176)$$

For No. 4,

$$H''' = \frac{wx'''}{8d} - \frac{wx'''^2}{8dl} - \frac{pwx'''}{4dl} + \frac{3p^2w}{8dl}. - (177)$$

An inspection of this compound truss will show that the compression in any member of the upper chord, QR for example, is the compression in that simple truss, to whose point in the lower chord R is opposite, or Simple Truss No. 1 at x, added to the strains in No. 4 at $x + p$, in No. 3 at $x + 2p$, and in No. 2 at $x - p$.

Making these changes in the values of x', x'', and x''', and then adding the equations, we obtain, after making $x = \frac{l}{2} - z$,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 + \frac{p^2w}{8dl}. \quad - \quad (178)$$

z is here the distance from the centre to the panel points in the upper chord of Simple Truss No. 3, and H is the strain in the member on the centre side of that point; and, similarly, the same equation will apply when z is made the distance to any panel point in the upper chord of No. 4.

By a similar process we obtain for the remaining members of the upper chord,

$$H' = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{p}{2}\right)^2 + \frac{9p^2w}{8dl}, \quad - \quad - \quad (179)$$

z' being the distance from the centre to any upper-chord point of No. 1 and No. 2; H′ being the compression on the centre side of that point.

In the lower chord,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 - \frac{15p^2w}{8dl}, \quad - \quad - \quad (180)$$

is the tension in those members on the centre side of the points in the same chord of Simple Trusses No. 3 and No. 4, and

$$H' = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{p}{2}\right)^2 - \frac{7p^2w}{8dl}, \quad - \quad - \quad (181)$$

is the tension in the members on the centre side of the points of No. 1 and No. 2.

These equations apply to all except the end members of both chords; their strains will be determined from the vertical equations.

130. Vertical Strains under a Full Load.—From the simple-truss horizontal equations (174), (175), (176), and (177), we have, for No 1,

$$V = \frac{w}{8} - \frac{wu}{4l}. \quad - \quad - \quad - \quad - \quad (182)$$

For No. 2,

$$V = \frac{w}{8} - \frac{w}{4l}(u' - p). \quad - \quad - \quad - \quad (183)$$

For No. 3,

$$V = \frac{w}{8} - \frac{wu''}{4l}. \quad - \quad - \quad - \quad - \quad (184)$$

And for No. 4,

$$V = \frac{w}{8} - \frac{w}{4l}(u''' + p). \quad - \quad - \quad (185)$$

131. Horizontal Strains in the end-chord Members.—The strains in the end members of either chord are the strains in the three simple trusses whose points come upon and next to the abutment, or, as that truss whose point is upon the abutment has no moment, the strain is equivalent to the strains in the two simple trusses whose points are next to the abutment. This strain is most easily obtained from the vertical equations, as was done in (100). Whence we obtain,

$$H = \frac{3pw}{4d} - \frac{p^2w}{dl}, \quad - \quad - \quad - \quad (186)$$

in the lower chord; and,

$$H = \frac{3pw}{8d}, \quad - \quad - \quad - \quad - \quad - \quad (187)$$

in the upper chord, when No. 4 has a point in the lower chord distant p from the abutment.

$$H = \frac{3pw}{8d} - \frac{3p^2w}{4dl}, \quad - \quad - \quad - \quad (188)$$

in the lower chord; and,

$$H = \frac{3pw}{8d} - \frac{p^2w}{4dl}, \quad - \quad - \quad - \quad - \quad (189)$$

in the upper chord, when No. 1 has a lower-chord point distant p from the abutment.

$$H = \frac{3pw}{8d} - \frac{3p^2w}{2dl}, \quad - \quad - \quad - \quad (190)$$

in the lower chord; and,

$$H = \frac{3pw}{8d} + \frac{p^2w}{2dl}, \quad - \quad - \quad - \quad (191)$$

in the upper chord, when No. 2 has a lower-chord point distant p from the abutment.

$$\mathrm{H} = \frac{3pw}{8d} - \frac{7p'w}{4dl}, \quad - \quad - \quad - \quad (192)$$

in the lower chord; and,

$$\mathrm{H} = \frac{3pw}{8d} + \frac{3p'w}{4dl}, \quad - \quad - \quad - \quad (193)$$

in the upper chord, when No. 3 has a lower-chord point distant p from the abutment.

132. Vertical Strains from a Moving Load.—The equations determined in the case of the quadruple truss with vertical struts and inclined ties are equally applicable to this truss, and their application is governed by the same principles.

133. Example.

In Fig. 52,

Let $l = 288$ feet, the length of the truss,

$d = 24$ feet, the depth of the truss,

$p = 12$ feet, the length of a panel or chord-member,

$w' = 288$ tons, the weight of a full movable load,

$w = 144$ tons, the constant-truss weight.

The load is upon the lower chord.

For the chord-members we use Eqs. (178), (179), (180), and (181), except for the ends, where Eqs. (186) and (187); w of these equations being equal to $w' + w$, or 432 tons; whence we can form the following table:

Values of z of Eqs. (179) & (180).	Values of z of Eqs. (178) & (181).	Strains in Tons.	Compression in	Strains in Tons.	Tension in
	12			639	mn & no
24		648	LM, MN, NO & OP	621	lm & op
36		630	KL & PQ	603	kl & pq
	48	594	IK & QR	585	ik & qr
	60	558	HI & RS	549	hi & rs
72		522	GH & ST	495	gh & st
84		468	FG & TU	441	fg & tu
	96	396	EF & UV	387	ef & uv
	108	324	DE & VW	315	de & vw
120		252	CD & WX	225	cd & wx
132		162	BC & XY	135	bc & xy
		81	AB & YZ	72	ab & yz

For the strains in the braces we have, for Simple Truss No. 4, Eq. (131) + Eq. (183),
For No. 3, Eq. (132) + Eq. (184),
For No. 2, Eq. (133) + Eq. (185), and
For No. 1, Eq. (134) + Eq. (182).

Substituting the values of the constants, and multiplying for the end braces by 1.118, and for all the others by 1.414, we have the following table:

Values of u in Eq. (133) & Eq. (185).	Values of u in Eq. (132) & Eq. (184).	Values of u in Eq. (131) & Eq. (184).	Values of u in Eq. (134) & Eq. (182).	Strains in Tons.	Compression in	Tension in
—12				60.38	Ab & Zy	Ba & Yz
	0			76.36	Ac & Zx	
		12		76.36	Bd & Yw	
			24	62.98	Ce & Xv	Ca & Xz
36				53.03	Df & Wu	Db & Wy
	48			52.32	Eg & Vt	Ec & Vx
		60		48.78	Fh & Us	Fd & Uw
			72	40.57	Gi & Tr	Ge & Tv
84				33.94	Hk & Sq	Hf & Su
	96			31.11	Il & Rp	Ig & Rt
		108		28.28	Km & Qo	Kh & Qs
			120	20.77	Ln & Pn	Li & Pr
132				14.85	Mo & Om	Mk & Oq
	144			12.73	Np & Nl	Nl & Np
		156		10.61	Oq & Mk	Om & Mo
			168	3.8	Pr & Li	Pn & Ln

Here Ln, Mo, Np, Oq, Pr, Pn, Om, Nl, Mk, and Li are subject to both tension and compression, or act as counterbraces. The compression in the end posts is the vertical strain in zy and zx, or 108 tons.

134.—It is not deemed necessary to give further examples of compound trusses with equally inclined struts and ties, the principles governing the determination of equations in any case having been sufficiently illustrated.

CHAPTER VI.

TRUSSES WITH HORIZONTAL CHORDS AND INCLINED BRACES, THE TIES HAVING A DIFFERENT INCLINATION FROM THE STRUTS, SUBJECT TO CONSTANT AND MOVING LOADS.

CASE I.—A DOUBLE TRUSS WHOSE UPPER CHORD IS DIVIDED INTO AN EVEN NUMBER OF PANELS OR MEMBERS.

135.—There is but one truss of the character described in the heading of this chapter in general use, the Post Truss, Fig. 54, where the struts have a horizontal extent of half that of the ties, the extent of the latter being equal to the depth of the truss.

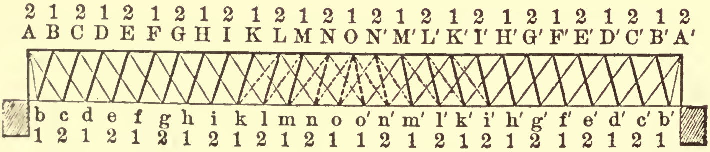


Fig. 54.

Under the action of the full load the truss may be resolved into two simple trusses, one of which is shown in Fig. 55, and termed, as in the previous cases, since its braces meet at the centre, No. 1.

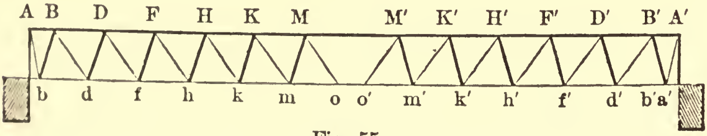


Fig. 55.

The other, shown in Fig. 56, is termed No. 2.

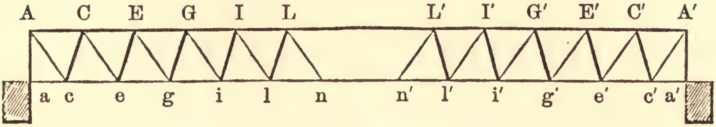


Fig. 56.

In both the counterbraces are removed.

136. Horizontal Strains.—Under a full load upon the lower chord, Simple Truss No. 1, having the panel point next the abutment, will be considered as bearing the load which comes directly upon the abutment, and which, since the point is only half a panel length from the abutment, is one-fourth a panel load. On the next to the abutment panel point rests a half panel load from one side, and a fourth panel load from the other, making upon it in all three-fourths a panel load.

Let l = the length of the truss,

d = the depth of the truss,

p = the length of a panel or any upper-chord member. The load, w, is upon the lower-chord.

Simple Truss No. 1 bears $\frac{w}{2}+\frac{wp}{l}$, and No. 2, $\frac{w}{2}-\frac{wp}{l}$ in this example.

Let x be the distance from the abutment to any point in the lower chord of No. 1; then the moment of the reaction of the abutment is $\frac{wx}{4}-\frac{wpx}{2l}$, the moment of the load directly upon the abutment, $\frac{wpx}{4l}$, of the load on the next panel point, $\frac{3pw}{4l}\left(x - \frac{p}{2}\right)$, and on the remainder of the truss $\frac{1}{2}$ of $\frac{w}{l}\left(x - \frac{5p}{2}\right)\left(x - \frac{p}{2}\right)$, whence we have,

$$H = \frac{wx}{4d} - \frac{wx^2}{4dl} + \frac{pwx}{4dl} + \frac{p^2w}{16dl}, \quad - \quad - \quad - \quad - \quad (194)$$

for the compressions in the upper-chord members opposite the panel points in the lower chord to which x is measured.

Taking moments around the panel points in the upper chord of No. 1, distant x' from the abutment, we obtain for the tensions in the lower-chord members of this simple truss opposite these points,

$$H' = \frac{wx'}{4d} - \frac{wx'^2}{4dl} + \frac{pwx'}{4dl} - \frac{p^2w}{8dl}. \quad - \quad - \quad (195)$$

From moments around the lower-chord points of Simple Truss No. 2, distant x'' from the abutment, the compressions in the upper-chord members opposite these points are given by

$$H'' = \frac{wx''}{4d} - \frac{wx''^2}{4dl} - \frac{pwx''}{4dl} + \frac{3p^2w}{16dl}. \quad - \quad - \quad (196)$$

And for the lower-chord members of the same truss, x''' being the distance to the upper-chord panel points,

$$H''' = \frac{wx'''}{4d} - \frac{wx'''^2}{4dl} + \frac{wpx'''}{4dl}. \quad - \quad (197)$$

The strain in the members of the upper chord of the double truss is found by making $\begin{Bmatrix} x'' \\ x \end{Bmatrix}$ of Eq. $\begin{Bmatrix} 196 \\ 194 \end{Bmatrix}$

equal to $\left\{\begin{matrix} x + p \\ x'' + p \end{matrix}\right\}$, and adding the equation so changed to Eq. $\left\{\begin{matrix} 194 \\ 196 \end{matrix}\right\}$, whence we have for the compressions in those members opposite the lower-chord points of No. 1, x having been made equal to $\frac{l}{2} - z$,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 - \frac{p^2 w}{8dl}. \quad - \quad (198)$$

And for the compressions in those members opposite the lower-chord points of No. 2, x'' having been made equal to $\frac{l}{2} - z'$,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' - \frac{p}{2}\right)^2 + \frac{3p^2 w}{8dl}, \quad - \quad (199)$$

z and z' being respectively the distances from the centre of the truss to the lower-chord panel points of Simple Trusses No. 1 and No. 2.

The tension in the lower chord of the double truss is found by making $\left\{\begin{matrix} x''' \\ x' \end{matrix}\right\}$ of Eq. $\left\{\begin{matrix} 197 \\ 195 \end{matrix}\right\}$ equal to $\left\{\begin{matrix} x' + p \\ x''' + p \end{matrix}\right\}$, and adding the equation so changed to Eq. $\left\{\begin{matrix} 195 \\ 197 \end{matrix}\right\}$; whence the strain opposite the upper-chord point of No. 1 is, making $x' = \frac{l}{2} - z$,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z + \frac{p}{2}\right)^2, \quad - \quad - \quad (200)$$

and opposite the upper-chord points of No. 2, making $x''' = \frac{l}{2} - z'$,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' + \frac{p}{2}\right)^2 - \frac{p^2 w}{2dl}. \quad - \quad (201)$$

137. Vertical Strains from a full Load.—As in the other compound trusses under a full load, the vertical strains here, in the simple trusses, are independent of each other, and may be obtained from the upper-chord simple-truss horizontal equations, and are,

$$V = \frac{w}{4} - \frac{w}{2l}\left(u - \frac{p}{2}\right), \quad - \quad - \quad - \quad (202)$$

for No. 1, u being the distance from the abutment to a point midway between the loaded points of this simple truss, and

$$V = \frac{w}{4} - \frac{w}{2l}\left(u' + \frac{p}{2}\right), \quad - \quad - \quad - \quad (203)$$

for No. 2, u' being the corresponding distance in this simple truss.

138. Vertical Strains from a Moving Load. — This truss has one decided peculiarity: the members of the simple trusses are connected by the counterbraces on the same side of the centre. Let the point l be loaded; then that portion of the weight which is borne by the right abutment is conveyed thither by the braces, through the points M, m, N, n, to O; from O it may pass to o, thence to N′ and thence solely through the braces of Simple Truss No. 2 to the abutment; or from O it may pass to o′, thence to M, and thence solely through the braces of No. 1 to the abutment; or again from O it may pass partly through o and partly through o′; but as the greatest strain must be provided for, each simple truss must be calculated to bear the whole of the strain which can possibly come upon it.

Let w' represent, as before, the weight of a full movable load, and let the partial load extend from one abutment a distance $(l - u)$ less than $\frac{l}{2}$; u in this case being the distance from the farther abutment to the centre of a panel of the double truss, the reaction of this abutment is, since the first panel point can bear only $\frac{3w'p}{4l}$,

$$V = \frac{w'}{2l^2}\left((l-u)^2 - \frac{p^2}{4}\right). \qquad - \quad - \qquad (204)$$

This would be the vertical strain from the partial load in the braces between the end of the load and the unloaded abutment, were there no strain from the constant-truss weight.

As this strain, Eq. (204), passes from the end of the load towards the centre, it meets the constant-truss strains, Eqs. (202) and (203), passing from the centre, the less of these neutralizing its amount in the greater; and since the moving-load strain passes through the counterbraces from one simple truss to the other, the same moving-load strain meets the constant-truss strain from both simple trusses, differing in this respect from its action in the other compound trusses.

Let every panel point, in the figure, from the left abutment to and including the point m, be fully loaded; then Eq. (204), when u is the distance from the right abutment to a point midway between m and n, is the portion of the load borne by the right abutment. A part of this strain at the point 1 meets the constant strain from Simple Truss No. 2, and the remainder of it meets the constant strain from No. 1 at m. If u and u' in the Eqs. (202) and (203) be made greater than $\frac{l}{2}$, and the difference in their successive values be kept constant, each represents the distances to the centre of the lower panels of the other simple truss than that to which it originally belonged, and V in each equation has a minus value, indicating a vertical strain passing from the abutment from which u and u' are measured. If u of one equation be made equal to $u' + p$ of the other, and the equations then added, we shall have,

$$\frac{w}{2} - \frac{w}{l}\left(u' + \frac{p}{2}\right), \quad - \quad - \quad - \quad (205)$$

for the total amount of constant-truss vertical strain that meets the strain of Eq. (204) when u' is greater than $\frac{l}{2}$; (had u' of the other equation been changed, and the two added, the resulting equation would be the same).

Eq. (205) has a minus value, consequently the difference between it and Eq. (204) may be obtained by adding the two; hence

$$V = \frac{w'}{2l^2}\left((l-u)^2 + \frac{p^2}{4}\right) + \frac{w}{2} - \frac{w}{l}\left(u' + \frac{p}{2}\right), \quad (206)$$

is, as long as it has a plus value, a strain passing to the abutment from which u is measured. When it has a minus value it passes to the other abutment and ceases to be of use, for it indicates a less strain than when the load extends to the same point from the other abutment.

The value of u' differs from that of u in this manner: Suppose the segment a to m, inclusive, to be loaded as before, then u' of the latter part of the equation is the distance from the right abutment to n, or the centre of a panel of a simple truss, but u is the distance from the same abutment to the centre of a panel of the double truss, in this case, to midway between m and n; hence $u = u' + \frac{p}{2}$, and the equation becomes

$$V = \frac{w'}{2l^2}\left((l-u)^2 - \frac{p^2}{4}\right) + \frac{w}{2} - \frac{wu}{l}. \quad (207)$$

This equation extends from that point where it first has a plus value to the centre, but no farther. When the load covers half the truss, the vertical strain to the unloaded abutment may take one of three courses, as explained before.

First, suppose it all to pass from the point o to

N′, or o′ to N, then since $u = \frac{l}{2}$, Eq. (204) becomes

$$V = \frac{w'}{8} - \frac{p^2 w'}{8l^2}, \qquad (208)$$

and this is the vertical strain throughout the braces of Simple Truss No. 2; then, as the load passes on, and the successive points of this simple truss become loaded, we have, for the load upon them, within the space $\frac{l}{2} - u$,

$$\frac{w'}{4l^2}\left((l-u')^2 - \frac{p^2}{4} - \frac{pl}{2}\right); \qquad (209)$$

adding this to Eq. (208), we have,

$$V = \frac{w'}{4l^2}(l-u')^2 + \frac{w'}{8} - \frac{3p^2w'}{16l^2} - \frac{w'p}{8l}. \qquad (210)$$

Again, adding to Eq. (203), we have,

$$V = \frac{w'}{4l^2}(l-u')^2 + \frac{w'}{8} - \frac{3p^2w'}{16l^2} - \frac{pw'}{8l} - \frac{w}{4} + \frac{w}{2l}\left(u' + \frac{p}{2}\right), \quad (211)$$

for the total vertical strain, or vertical component of the strain, in the braces of Simple Truss No. 2, when the load covers more than half the truss.

When the truss is half loaded, the strain from all except the point next the centre may pass through the braces of No. 1 to the unloaded abutment, or all of Eq. (204), when $u = \frac{l}{2} + p$, which is therefore,

$$V = \frac{w'}{8} + \frac{pw'}{2l} + \frac{p^2w'}{2l^2}. \qquad (212)$$

The load on the panel points of No. 1, in the space $\frac{l}{2} - u$, is,

$$V = \frac{w'}{16} - \frac{w'u}{4l} + \frac{u^2 w'}{4l^2} - \frac{p^2 w'}{16l^2}. \quad \cdot \quad \cdot \quad (213)$$

Adding these and Eq. (202), we have,

$$V = \frac{w'}{4l^2}(l-u)^2 + \frac{w'}{16} - \frac{3pw'}{8l} + \frac{3p^2 w'}{16l^2} + \frac{w}{4} - \frac{w}{2l}\left(u - \frac{p}{2}\right), (214)$$

for the maximum vertical strains in the braces of No. 1.

139. Example.—In Fig. 54,

Let l = 312 feet, the length of the truss,

d = 24 feet, the depth of the truss,

p = 12 feet, the length of a panel,

w' = 312 tons, the weight of a full movable load,

w = 156 tons, the weight of the truss.

For the horizontal chord strains, we use Eqs. (198), (199), (200), and (201). Substituting the values of the above constants, we can form the following table of strains:

Values of z in Eq. (198).	Values of z in Eq. (199).	Strains in Tons.	Compression in	Values of z in Eq. (200).	Values of z in Eq. (201).	Strains in Tons.	Tension in
6		759.375	NO & ON′	0		759.375	oo′
	18	759.375	MN& N′M′		12	745.875	no & o′n′
30		741.375	LM & M′L′	24		732.375	mn & n′m′
	42	723.375	KL & L′K′		36	700.875	lm & m′l′
54		687.375	IK & K′I′	48		669.375	kl & l′k′
	66	651.375	HI & I′H′		60	619.875	ik & k′i′
78		597.375	GH & H′G′	72		570.375	hi & i′h′
	90	543.375	FG & G′F′		84	502.875	gh & h′g′
102		471.375	EF & F′E′	96		435.375	fg & g′f′
	114	399.375	DE & E′D′		108	349.875	ef & f′e′
126		309.375	CD & D′C′	120		264.375	de & e′d′
	138	219.375	BC & C′B′		132	160.875	cd & d′c′
150		111.375	AB & B′A′	144		57.375	bc & c′b′

For the vertical strains, or strains having vertical components, we use Eq. (206) for the counterbraces when the load covers less than half the truss; Eq. (214) for the braces of No. 1, and Eq. (210) for the braces of No. 2, when the truss is more than half loaded. Each brace must be proportioned for the greatest strain which can come upon it in any case. The vertical strain is to be multiplied by 1.031 for the struts, and 1.25 for the ties, these being the secants of their angles. The following is a table of the strains in the braces:

Values of u in Eq. (214).	Values of u in Eq. (211).	Values of u in Eq. (206).	Strains in Tons.	Compression in	Strains in Tons.	Tension in
0					125.27	Ab & A′b′
	6				164.93	Ac & A′c′
18			124.70	Bb & B′b′	151.19	Bd & B′d′
	30		124.37	Cc & C′c′	150.79	Ce & C′e′
42			107.33	Dd & D′d′	125.13	Df & D′f′
	54		98.20	Ee & E′e′	119.06	Eg & E′g′
66			90.91	Ff & F′f′	110.22	Fh & F′h′
	78		82.26	Gg & G′g′	99.74	Gi & G′i′
90			75.37	Hh & H′h′	91.38	Hk & H′k′
	102		67.27	Ii & I′i′	81.56	Il & I′l′
114			60.93	Kk & K′k′	73.88	Km & K′m′
	126		53.23	Ll & L′l′	64.54	Ln & L′n′
138			47.37	Mm & M′m′	57.44	Mo & M′o′
		156	40.16	Nn & N′n′	48.71	No′ & N′o
		168			31.44	On′ & On
		180			19.83	N′m′ & Nm
		192			6.27	M′l′ & Ml

The compression in the end struts is 229.5 tons.

The ambiguity caused by the arrangement of the struts at the centre and the counterbraces may be entirely avoided, and the weight divided between the two simple trusses, by the arrangement shown in the following case:

CASE II.—A DOUBLE TRUSS WHOSE UPPER CHORD IS DIVIDED INTO AN ODD NUMBER OF PANELS OR MEMBERS.

140.—Let the Post Truss, described in the previous case, be opened at the centre, so that the upper ends of the centre struts are one panel length apart, and we have, adding the ties, the truss shown in Fig. (57).

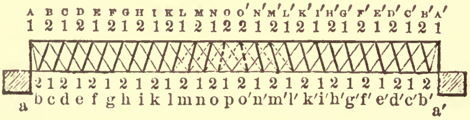


Fig. 57.

The struts and ties have the same inclinations as in the previous case.

141. Horizontal Strains.—Under the action of a full uniform load this truss may be resolved, as in previous cases, into two simple trusses, whose separate panel points are shown in the figure by the numbers 1, 1, etc., and 2, 2, etc.

Let l = the length of the truss,
d = the depth of the truss,
p = the length of a panel,
w = the weight of a full uniform load.

The load is upon the lower chord. As before, the quarter panel load, directly upon the abutment, is considered as belonging to that simple truss which has the panel point next the abutment. Therefore, in this example No. 1 bears $\frac{w}{2} - \frac{wp}{2l}$, and No. 2 bears $\frac{w}{2} + \frac{wp}{2l}$.

Hence, taking moments around any points of No. 1 in the lower chord, distant x from the abutment, we obtain for the strain in the upper chord opposite this point,

$$H = \frac{wx}{4d} - \frac{wx^2}{4dl} + \frac{3p^2w}{16dl}, \quad - \quad - \quad - \quad (215)$$

and for the lower chord, opposite any upper-chord panel point distant x' from the abutment,

$$H' = \frac{wx'}{4d} - \frac{wx'^2}{4dl}. \quad - \quad - \quad - \quad (216)$$

From moments around the points of No. 2 we have for the upper-chord member opposite any lower-chord point, distant x'' from the abutment,

$$H'' = \frac{wx''}{4d} - \frac{wx''^2}{4dl} + \frac{p^2w}{16dl}, \quad - \quad - \quad - \quad (217)$$

and for the lower-chord member, opposite any upper-chord point distant x''' from the abutment,

$$H''' = \frac{wx'''}{4d} - \frac{wx'''^2}{4dl} - \frac{p^2w}{8dl}. \quad - \quad - \quad - \quad (218)$$

These are the strains in the chords of the simple trusses under their separate loads. The strain in the upper chord of the double truss, opposite any lower-chord panel point, is the strain in the upper chord of the simple truss at the same point added to the strain in the other simple truss at the next panel point towards the centre; whence we obtain but one equation for the upper chord:

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z - \frac{p}{2}\right)^2 + \frac{p^2w}{8dl}, \quad - \quad - \quad (219)$$

z being $= \frac{l}{2} - x$, or the distance from the centre of the truss to the lower-chord panel point, opposite to which is the member whose strain is given by the equation, and is, therefore, the distance to the centre of that member.

In the lower chord of the double truss the strain at any point is the simple-truss strain at that point added to the other simple-truss strain at the next panel towards the abutment, whence we obtain for any member of the lower chord,

$$H = \frac{wl}{8d} - \frac{w}{2dl}\left(z' + \frac{p}{2}\right)^2 - \frac{p^2w}{4dl}, \quad - \quad - \quad (220)$$

z' being $= \frac{l}{2} - x$, or the distance from the centre of the truss to the centre of any lower-chord member whose strain is given by the equation.

142. Vertical Strain from a Constant Load.—This, as before, is obtained from the simple-truss horizontal equations, and is for Simple Truss No. 1,

$$V = \frac{w}{4} - \frac{wu}{2l}, \quad - \quad - \quad - \quad - \quad - \quad (221)$$

where u is the distance from the abutment to any point midway between the lower-chord points of No. 1; and for No. 2 it is,

$$V = \frac{w}{4} - \frac{wu'}{2l}, \quad - \quad - \quad - \quad - \qquad (222)$$

where u' is the distance from the abutment to any point midway between the lower-chord points of No. 2.

143. Vertical Strains from a Moving Load.—If we consider the truss as half loaded, it will be seen from the figure that the portion of the weight borne by the unloaded abutment, excepting that upon the central panel point, passes from the end of the load to that abutment solely through the braces of Simple Truss No. 2. If less than half the truss be loaded, the load extending from one end, the weight borne by the unloaded abutment passes by means of the counterbraces from one simple truss to the other until it reaches the centre, beyond which it is confined to No. 2, as before.

In the case of a less than half load, the conditions are similar to the previously-described Post Truss, and Eq. (204),

$$V = \frac{w'}{2l^2}\left((l-u)^2 - \frac{p^2}{4}\right),$$

gives the vertical strain passing to the abutment from which u is measured, the minimum value of u being $\frac{l}{2} + \frac{p}{2}$, and w' being the weight of the full load.

To this strain must be added the equation of the constant-truss strain. This is Eq. $\begin{Bmatrix}221\\222\end{Bmatrix}$ added to Eq. $\begin{Bmatrix}222\\221\end{Bmatrix}$ when $\begin{Bmatrix}u'\\u\end{Bmatrix}$ is made equal to $\begin{Bmatrix}u+p\\u'+p\end{Bmatrix}$. Whence,

$$V = \frac{w}{2} - \frac{w}{l}u)' + \frac{p}{2}\Big); \quad - \quad - \quad - \quad (223)$$

and since $u' + \frac{p}{2}$ of this equation is the distance to the same point as u of Eq. (204), we have, adding these equations,

$$V = \frac{w'}{2l^2}\left((l-u)^2 - \frac{p^2}{4}\right) + \frac{w}{2} - \frac{wu}{l}. \quad - \quad (224)$$

It must be borne in mind that u, in this equation, is the distance from the abutment to the centre of a panel of the double truss, and that its least value is $\frac{l}{2} + \frac{p}{2}$, and that the equation is only needed as long as V has a plus value.

When the load extends from one abutment beyond the centre, we have for the strains on the braces of No. 2, the strain of Eq. (204), when $u = \frac{l}{2} + \frac{p}{2}$, or,

$$V = \frac{w'}{8} - \frac{w'p}{4l}. \quad - \quad - \quad - \quad - \quad - \quad (225)$$

Then, as the successive points of No. 2 become loaded, we have, for the proportion of weight passing to the unloaded abutment from which u' is measured,

$$\frac{w'}{4l}\left(\frac{l}{2} - u'\right) + \frac{w}{4l^2}\left(\frac{l}{2} - u'\right)^2,$$

which is to be added to Eq. (225), and to the constant-truss strain of No. 2, Eq. (222), and we have,

$$V = \frac{w'}{4l^2}(l - u')^2 + \frac{w'}{16} - \frac{w'p}{4l} + \frac{w}{4} - \frac{wu'}{2l}, \quad (226)$$

for the maximum strain upon the braces of No. 2, in that panel whose centre is distant u' from the abutment.

Until the moving load, extending from one abutment, reaches the central panel point, no vertical strain from it comes upon the braces of No. 1 in the unloaded half; the greatest strain upon these braces from the central point is when it has a panel load upon it. As the successive points towards the unloaded abutment come under the load, we have for the proportion of weight borne by this abutment and the greatest vertical strain from the movable load in the panel of No. 1 to which u is measured,

$$V = \frac{w'}{4l^2}(l-u)^2 - \frac{w'}{4l^2}\left(\frac{l}{2} - p\right)^2; \quad (227)$$

and adding this to Eq. (221), we have,

$$V = \frac{w'}{4l^2}(l-u)^2 - \frac{w'}{4l^2}\left(\frac{l}{2} - p\right)^2 + \frac{w}{4} - \frac{wu}{2l}, \quad (228)$$

for the maximum strain in No. 1.

There is a portion of the load on No. 1, between the centre and the end of the load, that passes to the other abutment, or the one from which the load extends, the strain of which is taken back to the centre by the counterbraces.

144. Example.—In Fig. (57),

Let l = 324 feet, the length of the truss,

d = 24 feet, the depth of the truss,

p = 12 feet, the length of a panel,

w' = 324 tons, the weight of a full movable load,

w = 162 tons, the weight of the uniform permanent load.

145. Horizontal Strains.—Substitute the values of the constants in Eqs. (219) and (220), w of these equations being $w' + w$, or 486 tons, and we have the following table of strains:

Values of z in Eq. (219).	Strains in Tons.	Compression in	Values of z' in Eq. (220).	Strains in Tons.	Tension in
0	820.125	OO′	6	813.375	op & po′
12	820.125	NO & N′O′	18	779.875	no & o′n′
24	811.125	MN & M′N′	30	777.375	mn & n′m′
36	793.125	LM & L′M′	42	745.875	lm & m′l′
48	766.125	KL & K′L′	54	705.375	kl & l′k′
60	730.125	IK & I′K′	66	655.875	ik & k′i′
72	685.125	HI & H′I′	78	596.375	hi & i′h′
84	631.125	GH & G′H′	90	529.875	gh & h′g′
96	568.125	FG & F′G′	102	453.375	fg & g′f′
108	496.125	EF & E′F′	114	367.875	ef & f′e′
120	415.125	DE & D′E′	126	273.375	de & e′d′
132	325.125	CD & C′D′	138	171.875	cd & d′c′
144	226.125	BC & B′C′	150	57.375	bc & c′b′
156	118.125	AB & A′B′			

146. Vertical Strains.—Substitute the values of the constants in Eqs. (228), (226), and (224), and multiplying the results by 1.25 for the ties, and 1.031 for the struts, we have the following table of strains in the braces:

Values of u in Eq. (224).	168	180	192	204			
Strains in Tons.		23.67	14.36	1.20			
Compression in		Oo & O′o′	Nn & N′n′	Mm & M′m′			
Strains in Tons.	43.13	28.70	17.41	1.45			
Tension in	oO & o′O	nO & n′O′	mN & m′N′	lM & l′M′			
Values of u in Eq. (226).	150	126	102	78	54	30	6
Strains in Tons.	9.28	22.57	36.78	51.89	67.93	84.88	
Compression in	Nn & N′n′	Ll & L′l′	Ii & I′i′	Gg & G′g′	Ee & E′e′	Cc & C′c′	
Strains in Tons.	11.25	27.36	44.59	62.91	82.36	102.91	124.59
Tension in	Np & N′p	Ln & L′n′	Il & I′l′	Gi & G′i′	Eg & E′g′	Ce & C′e′	Ac & A′c′
Values of u in Eq. (228).	138	114	90	66	42	18	0
Strains in Tons.	51.49	65.24	79.90	95.47	111.98	129.39	
Compression in	Mm & M′m′	Kk & K′k′	Hh & H h′	Ff & F′f′	Dd & D′d′	Bb & B′b′.	
Strains in Tons.	62.43	79.10	96.88	115.75	135.76	156.88	125.27
Tension in	Mo & M′o′	Km & K′m′	Hk & H′k′	Fh & F′h′	Df & D′f′	Bd & B′d′	Ab & A′b′

Total compression on the end struts, 238.5 tons.

The compression on Nn and N′n′ is greater in the second equation than in the first.

147.—This system of bracing, like the others, may be extended to compound trusses containing any number of simple trusses, for which equations may be similarly found.

CHAPTER VII.

INCLINED TRUSSES OR RAFTERS.

148.—The same principles govern the investigation of strains in inclined as in horizontal trusses, and moments are to be taken and equations formed in the same manner.

The inclined truss thrusts horizontally as well as vertically against its abutments or walls, or the latter may be considered as having a horizontal reaction inward as well as a vertical reaction upward.

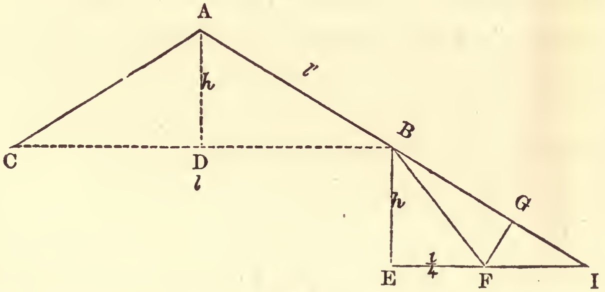


Fig. 58.

149. Horizontal Reaction.—Let AB and AC, Fig. 58, represent a pair of inclined trusses or rafters, joined together at the apex and to the walls by one chord only.

Let l = the horizontal distance, BC, between the walls,

l' = the length of each rafter,

h = the height of the apex, A, above the walls,

w = the whole weight, uniformly distributed over the two rafters.

The rafter to the right of A is kept in equilibrium by the vertical reaction of the wall, $\frac{w}{2}$, the weight on the rafter, $\frac{w}{2}$, and the inward reaction of the wall; hence, taking moments around A, we have for the moment of the latter, the difference between the moments of the other forces, or,

$$Hh = \frac{w}{2} \times \frac{l}{2} - \frac{w}{2} \times \frac{l}{4},$$

$$\because H = \frac{wl}{8h}, \quad - \quad - \quad - \quad - \quad - \qquad (229)$$

is the horizontal reaction of either wall.

Taking moments around D, in the line of the tops of the walls, it will be found that the horizontal thrust at A is the same.

150.—Had the rafters been joined by both chords at A, it would be impossible to determine the proportion of strain to which each chord would then become subject. In such a case one rafter could not bear an equal or certain fixed proportion of strain to the other unless the mechanical construction possessed a theoretical precision almost unattainable in practice; and even were this done, the different expansions and contractions of the two chords would immediately affect the amount of strain upon each.

All ambiguity of strains in trusses must, where practicable, be avoided, and we shall therefore consider the rafters as joined together by one chord only.

151. The proportion between the Horizontal and Vertical Reactions of the Wall.—To find the proportion between the horizontal and vertical reactions of the wall, we have,

$$\frac{wl}{8h} : \frac{w}{2} :: h : \frac{l}{4}.$$

Draw, in Fig. 58, the vertical line BE $= h$, and the horizontal line EF $= \frac{l}{4}$. If BE represent the amount of vertical reaction, or $\frac{w}{2}$, then EF will represent the horizontal reaction, and this proportion will hold good for any value of w, great or small.

The resultant force of BE and EF is represented by the line BF. This line gives the direction and amount of thrust of the rafter, and BE and EF are the vertical and horizontal components of this thrust.

It will be noticed that the resultant thrust is not in the line of the rafter, and that its direction depends on the inclination of the rafter and not on w.

152. Longitudinal Reaction.—Instead of horizontal and vertical components, this thrust of the rafter, represented by the line BF, may be resolved into two other components, one shown by BA in the line of the rafter, and the other by GF at right angles to it. The first of these we shall term the Longitudinal Reaction, and the other the Transverse Reaction, the value of the former being found as follows: Produce the lines EF and BG until they meet at I, and from F draw FG parallel to BI; therefore $FI = FE = \frac{l}{4}$, and the triangle FGI is similar to the triangle ADB, and

$$l' : \frac{l}{2} :: \frac{l}{4} : \frac{l^2}{8l'} = \text{the length of GI.}$$

Then,

$$h : l' - \frac{l^2}{8l'} :: \frac{w}{2} : \frac{wl'}{2h} - \frac{wl^2}{16hl'},$$

and,

$$\frac{l^2}{4} = l'^2 - h^2,$$

$$\therefore \quad \frac{wl'}{2h} - \frac{wl^2}{16hl'} = \frac{wl'}{2h} - \frac{w}{4hl'}(l'^2 - h^2) = \frac{wl'}{4h} + \frac{wh}{4l'},$$

or,
$$L = \frac{wl'}{4h} + \frac{wh}{4l'}, \qquad (230)$$

is the longitudinal reaction of the wall or abutment.

153. Transverse Reaction.—The transverse reaction is found by a similar process.

$$l' : h :: \frac{l}{4} : \frac{hl}{4l'} = \text{the distance GF,}$$

and

$$h : \frac{hl}{4l'} :: \frac{w}{2} : \frac{wl}{8l'};$$

or,

$$T = \frac{wl}{8l'}, \qquad (231)$$

is the transverse reaction of the wall.

154. Truss Strains.—Let Fig. 59 represent a pair of rafters,

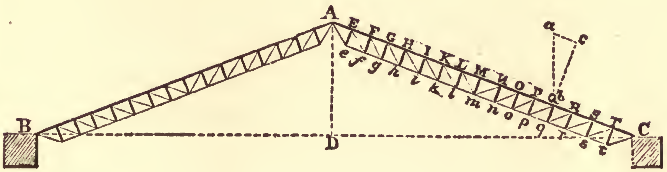


Fig. 59.

or inclined trusses, joined to each other and to the wall by the upper chord, and divided into uniform panels by the braces.

Let w = the weight, uniformly distributed over the two rafters.

l = the distance between the walls or lower ends of the rafters.

l' = the length of each rafter between the points of its connection.

h = the height of the apex above the supports, or the height of one end of a rafter above the other.

d = the depth of the rafters, or the distance between the chords at right angles to the lengths.

x = the distance along the upper chord to any panel point from the point where it is joined to the abutment.

L = the longitudinal chord strain.

T = the transverse strain.

The weight is considered as concentrated at the upper-chord panel points—the abutment bearing half a panel load.

155. Longitudinal Strains.—If we take moments around any panel point in the upper chord distant x from the wall, the moment of the longitudinal strain in the lower chord, in the same transverse section, is $L \times d$; and it is equal to the moment of the transverse reaction of the abutment, less the moment of the transverse component of the load on x: the longitudinal reaction of the abutment, and the longitudinal component of the weight on x, passing through the point about which moments are taken, have no moment.

The weight on x is $\frac{w}{2l'} \times x$; and if it be represented by the line ab in Fig. 59, bc will be its transverse, and ac its longitudinal component, and the triangle abc will be similar to the triangle ACD; hence we obtain the former component from the proportion

$$l' : \frac{l}{2} :: \frac{wx}{2l'} : \frac{wlx}{4l'^2},$$

and its moment is $\frac{wlx}{4l'^2} \times \frac{x}{2} = \frac{wlx^2}{8l'^2}$; whence we can form the equation

$$L = \frac{wlx}{8dl'} - \frac{wlx^2}{8dl'^2}, \quad - \quad - \quad - \quad - \qquad (232)$$

for the longitudinal tension in the lower chord opposite any panel point distant x from the wall.

Taking moments around a point in the lower chord in the same transverse section, the moment of the transverse reaction of the wall and the load on x are the same as in the other chord. In this case the conditions, so far as the moment of the load on x is considered, are the same as those which in (28) gave Eq. (14).

We have, therefore, Eq. (232) for the upper and lower chord strains, and making $x = \frac{l}{2} - z$, we obtain,

$$L = \frac{wl}{32d} - \frac{wlz^2}{8dl'^2}, \qquad (233)$$

where z is the distance from the centre of the upper chord to the different panel points.

This strain, Eq. (233), is from the transverse reaction of the abutment; but other strains from the longitudinal component of the weight upon the rafter, and the longitudinal reaction of the abutment, affect the upper chord.

In considering the effect of these strains it will be noticed that, though the weight on x is $\frac{wx}{2l'}$, a half-panel load rests directly on the abutment, and consequently that $\frac{w}{2l'}\left(x - \frac{p}{2}\right)$ will express the true amount of the load between the panel point to which x is measured and the abutment. The moment of its longitudinal component is $\frac{wdh}{2l'^2}\left(x - \frac{p}{2}\right)$, and the moment of the longitudinal reaction is, from Eq. (230), $\left(\frac{wl'}{4h} + \frac{wh}{4l'}\right)d$, whence

$$Ld = \left(\frac{wl'}{4h} + \frac{wh}{4l'}\right)d - \frac{wdh}{2l'^2}\left(x - \frac{p}{2l'}\right)$$

and

$$L = \frac{wl'}{4h} + \frac{wh}{4l'} - \frac{wh}{2l'^2}\left(x - \frac{p}{2}\right), \quad - \quad - \quad (234)$$

is the compression from the longitudinal strains which are confined to the upper chord; and for this chord is to be added to Eq. (233); bearing in mind that Eq. (234) applies throughout the truss to the member on the abutment or lower side of the point to which x is measured, and Eq. (233) to the member, as will be seen hereafter, on the side on which there is no inclined brace, of the point to which z is measured; the equations have been separated to avoid confusion from this cause.

Eq. (232) is greatest when $x = \frac{l}{2}$, or at the centre, when it becomes

$$L = \frac{wl}{32d}, \quad - \quad - \quad - \quad - \quad - \quad - \quad (235)$$

and decreases both ways to the ends, where it becomes zero.

Eq. (234) varies inversely as x, and is, consequently, greatest at the abutment and least at the apex; its difference between any two points distant p apart, that is, any two contiguous members, is, it will be seen by subtracting Eq. (234) at x from Eq. (234) when x is made $x + p$, $\frac{wph}{2l'^2}$, or the longitudinal component of a panel load. It therefore receives no strains from the braces, nor transmits any to them, but is confined wholly to the upper chord.

156. Transverse Strains.—As Eq. (232) increases as we pass from either end towards the centre of the rafter, this change of strain must affect the braces, and as Eq. (232) is similar to the horizontal equations for the horizontal trusses, the braces, it will be seen, will depend upon their inclinations for their character of struts and ties; that is, if they are alternately transverse and diagonal, the latter will be ties when their upper or outer ends are inclined towards the ends of the rafter, and the former, struts; the reverse being also true. Similarly to the other trusses, the difference in the different values of Eq. (232) at the two ends of a panel is the longitudinal component of the strain in the tie of that panel. (Longitudinal, in this chapter, refers exclusively to line of direction of the chord).

The difference in the values of Eq. (232) at x and at $x-p$, is,

$$\frac{wlp}{8dl'} - \frac{wlp}{4dl'^2}\left(x - \frac{p}{2}\right),$$

and from the proportion,

$$d : p :: \frac{wlp}{8dl'} - \frac{wlp}{4dl'^2}\left(x - \frac{p}{2}\right) : \frac{wl}{8l'} - \frac{wl}{4l'^2}\left(x - \frac{p}{2}\right)$$

and making $u = x - \frac{p}{2}$, or the distance from the end to the centre of a panel, we have,

$$T = \frac{wl}{8l'} - \frac{wlu}{8l'^2}. \qquad - \quad - \quad - \quad - \qquad (236)$$

This strain becomes zero when $u = \frac{l'}{2}$, and increases towards either end; but when u becomes greater than $\frac{l'}{2}$, T has a minus value, indicating a strain passing to the opposite end to that from which u is measured.

157.—At either end of the rafter,

$$T = \frac{wl}{8l'}. \qquad (237)$$

At the upper end, where $x = l'$, Eq. (232) becomes zero, and Eq. (234), putting l' for $x - \frac{p}{2}$, becomes

$$L = \frac{wl'}{4h} - \frac{wh}{4l'}. \qquad (238)$$

Hence we have two strains at the apex, one, Eq. (238), thrusting upward and against the other rafter; the other thrusting downward and also against the other rafter. The horizontal component of Eq. (238) is found from the proportion,

$$l' : \frac{l}{2} :: \frac{wl'}{4h} - \frac{wh}{4l'} : \frac{wl}{8h} - \frac{whl}{8l'^2};$$

and of Eq. (237) from the proportion,

$$l' : h :: \frac{wl}{8l'} : \frac{whl}{8l'^2},$$

adding, we have,

$$H = \frac{wl}{8h} - \frac{whl}{8l'^2} + \frac{whl}{8l'^2} = \frac{wl}{8h}, \qquad (239)$$

total horizontal strain at the apex.

The vertical component of Eq. (238) is from the proportion, $l' : h :: \frac{wl'}{4h} - \frac{wh}{4l'} : \frac{w}{4} - \frac{wh^2}{4l'^2}$, a thrust upward.

The vertical component of Eq. (237) is from the proportion,

$$l' : \frac{l}{2} :: \frac{wl}{8l'} : \frac{wl^2}{16l'^2},$$

and since $\frac{l^2}{4} = l'^2 - h^2$, $\frac{wl'^2}{16l'^2} = \frac{w}{4} - \frac{wh^2}{4l'^2}$,

a thrust downward, equal in amount to the other and neutralizing it, leaving only a horizontal strain at the apex.

158. Example.—Let Fig. 59 represent a pair of rafter trusses;

Let l = 150 feet, the distance between the lower ends of the rafters,

l' = 80 feet, the length of either rafter,

h = 27.84 feet, the height of the apex above the wall,

p = 5 feet, the length of a panel,

d = 2.5 feet, the transverse depth of either truss,

w = 20 tons, the weight, distributed uniformly over the trusses.

159. Longitudinal Strains. — For the upper-chord strains substitute values in Eq. (233), the first value of z being zero, and in Eq. (234), and add the strains so found; for the lower chord use only Eq. (233), the first value of z being 5. In the table of chord-strains below, the strains for the upper chord, from the two equations, are given separately and then added.

Values of z in Eq. (233).	Strains in Tons.	Tension in	Compression in	Values of x in Eq. (234).	Strains in Tons.	Total upper-chord compression.
35	8.79	st	TC	5	16.00	24.79
30	16.41	rs	ST	10	15.78	32.19
25	22.86	qr	RS	15	15.57	38.43
20	28.12	pq	QR	20	15.35	43.47
15	32.20	op	PQ	25	15.13	47.33
10	35.16	no	OP	30	14.91	50.07
5	36.91	mn	NO	35	14.70	50.61
0	37.5		MN	40	14.48	51.98
0	37.5		LM	45	14.26	51.76
5	36.91	lm	KL	50	14.04	50.95
10	35.16	kl	IK	55	13.82	48.98
15	32.20	ik	HI	60	13.60	45.80
20	28.12	hi	GH	65	13.39	41.51
25	22.86	gh	FG	70	13.17	36.03
30	16.41	fg	EF	75	12.95	29.36
35	8.79	ef	AE	80	12.73	21.52

160. Transverse Strains.—The load being concentrated upon the upper-chord points, the strut belongs to that point to which u is measured; Eq. (236) gives the strains in the struts, and multiplied by 2.236, the secant of the angle made by the ties with the struts, gives the strains in the ties; whence, by substituting the values given above, we can form the following table:

Values of u.	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5
Strains in Tons.	4.39	3.81	3.22	2.64	2.05	1.46	.88	.58
Compression in	Tt & Ee	Ss & Ff	Rr & Gg	Qq & Hh	Pp & Ii	Oo & Kk	Nn & Ll	Mm
Strains in Tons.	9.82	8.52	7.20	5.90	4.58	3.26	1.97	.65
Tension in	Ae & tC	Ef & sT	Fg & rS	Gh & qR	Hi & pQ	Ik & oP	Kl & nO	Lm & mN

161. An Inclined Truss whose Struts and Ties make the same Angle with the Chord.—If the truss shown in Fig. (48) be used as a rafter, the longitudinal and transverse reactions of the wall remain the same; the longitudinal component of the weight on any segment remains the same, and Eq. (234) will apply without change; the moments for the lower chord are taken similarly, so that Eq. (232) holds for that also. But Eq. (232) is adapted, in this case, to the upper chord, because x must be measured to the lower-chord panel points, instead of to the loaded points.

The moment of the weight on the segment x' is in this case $\frac{wx'}{4dl'} + \frac{wp^2}{4dl'}$,

and its transverse component is given by the proportion,

$$l' : \frac{l}{2} :: \frac{wx'}{4dl'} + \frac{wp^2}{4dl'} : \frac{wlx^2}{8dl'^2} + \frac{wlp^2}{8dl'^2};$$

therefore Eq. (232) for the upper chord becomes

$$L = \frac{wlx'}{8dl'} - \frac{wlx'^2}{8dl'^2} - \frac{wlp^2}{8dl'^2}, \quad - \quad - \quad - \quad (240)$$

x' being the distances to the lower-chord points.

In this case vertical equation (236) applies without change, u being the distance to the centre of any upper-chord panel member. The struts and ties are distinguished by their inclinations, and are the same as in the same truss placed horizontally.

CHAPTER VIII.

TRIANGULAR TRUSSES.

162.—Triangular Trusses are those in which one horizontal chord forms the base of an isòsceles triangle, the remaining two sides acting as the other chord, with braces between. They are generally used to support roofs, but also possess many advantages for bridge purposes.

163.—Let AB and BC, Fig. 60, represent two rafters whose thrust at their supports is taken by the horizontal chord BC, and connected to the chord by braces, omitted in the figure.

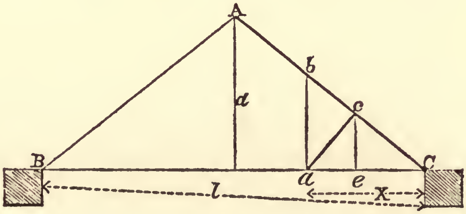


Fig. 60.

ABC is a triangular truss, AB and AC acting as one chord, the strains in which will be termed the longitudinal strains.

164. Longitudinal Strains.—

Let w = the weight, uniformly distributed over the chord struts, AB and BC,

l = the horizontal distance between the abutments, B and C,

l' = the length of either chord strut,

d = the height of the apex above the abutments, or the depth of the truss at the centre,

x = the horizontal distance from one of the abutments to any point between that abutment and the centre,

H = the horizontal strain in the horizontal chord,

L = the longitudinal strain in the chord struts,

V = the vertical strain, or vertical component of the strain affecting the braces.

Disregarding for the present the braces, and taking moments around a, in a vertical section cutting no inclined brace, distant x from the abutment C, that part of AC to the right of ab is kept in equilibrium by the reaction of the right abutment, $\frac{w}{2}$, the load on bc, $\frac{wx}{l}$, and the longitudinal strain at b.

Hence L, the longitudinal strain at b, multiplied by ac, the distance from a at right angles to its direction; or,

$$L \times ac = \frac{wx}{2} - \frac{wx^2}{2l}.$$

From similar triangles,

$$l' : d :: x : ac, \text{ or } ac = \frac{dx}{l'}.$$

Substituting, we have,

$$L\frac{dx}{l'} = \frac{wx}{2} - \frac{wx^2}{2l},$$

and

$$L = \frac{wl'}{2d} - \frac{wl'x}{2dl}. \quad - \quad - \quad - \quad - \quad (241)$$

From the shape of this truss it is evident that the maximum value of x is $\frac{l}{2}$, and that its limits are zero and $\frac{l}{2}$; when it is zero,

$$L = \frac{wl'}{2d}, \quad - \quad - \quad - \quad - \quad - \quad (242)$$

the longitudinal strain at the abutment, whose vertical component, found from the proportion $l' : d$, is $\frac{w}{2}$; or at the abutment the chord strut contains the whole vertical strain. When $x = \frac{l}{2}$,

$$L = \frac{wl'}{4d}, \quad - \quad - \quad - \quad - \quad - \quad - \quad (243)$$

and its horizontal component, from the proportion $l' : \frac{l}{2}$, is $\frac{wl}{8d}$.

165. Horizontal Strains.—Taking moments around b, those of the load on bc and the reaction of the abutment remain the same as above, and we have for the strain at a,

$$H \times (ab) = \frac{wx}{2} - \frac{wx^2}{2l},$$

and $\frac{l}{2} : d :: x : \frac{2dx}{l}$, the distance ab.

Substituting, we have,

$$H\frac{2dx}{l} = \frac{wx}{2} - \frac{wx^2}{2l},$$

and

$$H = \frac{wl}{4d} - \frac{wx}{4d}. \quad - \quad - \quad - \quad - \quad - \quad (244)$$

This strain, like the longitudinal, is greatest at the abutment, where $x = 0$, and

$$H = \frac{wl}{4d}, \quad - \quad - \quad - \quad - \quad - \quad (245)$$

and least at the centre, where $x = \frac{l}{2}$, and,

$$H = \frac{wl}{8d}, \quad - \quad - \quad - \quad - \quad - \quad - \quad (246)$$

or it equals the strain in a horizontal truss at the centre, and increases from that point to the abutment, where it is double that amount.

166. Vertical Strains.—H of Eq. (244) being the horizontal strain at a;

$$H' = \frac{wl}{4d} - \frac{wx'}{4d},$$

will be the horizontal strain at the point d, distant x' from the same abutment. The strain at d is evidently greater than that at a; subtracting,

$$H' - H = \frac{w}{4d}(x - x'),$$

is the difference; this may be caused by a tie from d to b, with a strut from b to a, which would add to the tension at d; or a strut from a to c, with a vertical tie cd, which would neutralize a certain amount of tension at a. Let the latter case be supposed; then, since $\frac{w}{4d}(x - x')$ is the horizontal component of the compression in ac, its vertical component is found from the proportion $ad : dc$, or,

$$x - x' : \frac{2dx'}{l} :: \frac{w}{4d}(x - x') : \frac{wx'}{2l}.$$

Hence,

$$V = \frac{wx'}{2l}, \qquad - \quad - \quad - \quad - \quad - \qquad (247)$$

is the vertical component of the strain in the brace ac.

Now, if the truss be divided into panels of uniform horizontal length, braced as in Fig. (62), with inclined struts and vertical ties, and the weight considered as concentrated at the upper panel points, it will still perfectly fulfil the conditions on which the above equations have been based; and Eq. (247) is the vertical component of the strain in that strut whose upper end is distant x horizontally from the abutment. It will be noticed that Eq. (247), or the vertical strain in the braces, increases from zero at the abutment, where $x = 0$, to $\frac{w}{4}$ at the centre, where $x = \frac{l}{2}$.

Attention must be directed to the difference between the equations of the triangular and of the horizontal trusses.

The tension in any vertical tie is plainly the vertical strain in that strut to whose lower end it is attached, except the centre tie, which receives the vertical strain from two struts.

167. Example.—In Fig. (61),

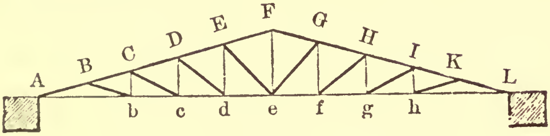


Fig. 61.

Let $w = 50$ tons, whole weight,

$l = 100$ feet, the length of the truss,

$l' = 51.4$ feet, the length of the chord struts,

$d = 12.5$ feet, the height of the truss at the centre,

$p = 10$ feet, the horizontal length of a panel.

From Eqs. (241) and (244) we have the following table of the longitudinal and horizontal strains :

Values of x.	10	20	30	40	50
Strains in Tons.	92.56	82.27	71.99	61.70	51.49
Compression in	AB & KL	BC & IK	CD & HI	DE & GH	EF & FG
Strains in Tons.	90	80	70	60	
Tension in	Ab & hL	bc & gh	cd & fg	de & ef	

From Eq. (247) we obtain the vertical strains and the compression in the struts from the proportion of the vertical height of the strut to its length, or what is the same, multiplying the vertical strain by the secant of the angle of the strut and a vertical.

Values of x.	10	20	30	40
Strains in Tons.	10.31	11.18	12.5	14.14
Compression in	Bb & Kh	Cc & Ig	Dd & Hf	Ee & Ge
Strains in Tons.	2.5	5	7.5	20
Tension in	Cb & Ih	Dc & Hg	Ed & Gf	Fe

168. Inclined Struts and Ties.—If a triangular truss be braced as in Fig. (62), the lower

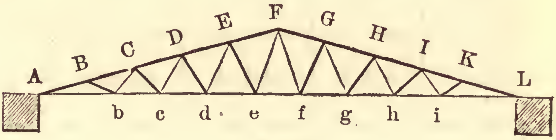


Fig. 62.

chord points are not vertically beneath the upper points, and the longitudinal equation becomes changed; and

$$L\frac{dx}{l'} = \frac{wx}{2} - \frac{wx^2}{2l} + \frac{wp^2}{2l},$$

whence,

$$L = \frac{wl'}{2d} - \frac{wl'x}{2dl} + \frac{wl'p^2}{2dlx}. \quad - \quad - \quad - \quad (248)$$

The vertical equation, $\frac{wx}{2l}$, remains the same, x being still the horizontal distance to the upper end of any strut; the tie connected to the lower end of any strut having the same vertical strain as that strut. If the weight and dimensions of Fig. (62) be assumed the same as those of Fig. (61), we have the following table of longitudinal and horizontal strains:

Values of x.	5	15	25	35	45
Strains in Tons	92.56	85.71	76.11	66.13	56.01
Compression in	AB & KL	BC & IK	CD & HI	DE & GH	EF & FG
Values of x.	10	20	30	40	50
Strains in Tons.	90	80	70	60	50
Tension in	Ab & iL	bc & hi	cd & gh	de & fg	ef

And the following table of strains in the braces:

Values of x.	10	20	30	40
Strains in Tons.	5.59	7.07	9.01	11.18
Compression in	Ki & Bb	Cc & Ih	Dd & Hg	Ee & Gf
Strains in Tons.	3.53	6.00	8.39	10.76
Tension in	Cb & Ii	Dc & Hh	Ed & Gg	Fe & Ff

169. The Triangular Truss subject to Movable Loads.—When used for bridge purposes the load will be upon the horizontal chord, and generally the truss will be inverted, in which case the longitudinal and horizontal strains will remain the same in amount, though changed in character; the vertical equation will also remain the same.

We shall now consider the effects of movable loads to which the truss is subject when used as a bridge. Let the truss, Fig. (62) inverted, be partially, but more than half loaded, let the load extend from the left abutment to a point distant u from the right abutment, and let w' = the weight of a full load of uniform density with the partial load. By the principles of the lever, $\frac{w'}{2l^2}(l - u)^2$ is the reaction of the right abutment. Taking moments around a point in the chord tie, or the lower chord, distant x horizontally from the right abutment, x being less than u, we have for the strain in the horizontal chord,

$$H \frac{2dx}{l} = \frac{w'x}{2l^2}(l - u)^2,$$

and,

$$H = \frac{w}{4dl}(l - u)^2, \quad - \quad - \quad - \quad - \quad (249)$$

a constant for the entire distance u.

In the same manner for the tension in the lower chord,

$$L \frac{dx}{l'} = \frac{wx}{2l^2}(l - u)^2,$$

and

$$L = \frac{wl'}{2dl}(l - u)^2, \quad - \quad - \quad - \quad - \quad (250)$$

also a constant, or, the compression in the upper chord and the tension in the lower chord are the same from the end of the partial load to the abutment. Consequently no strain comes upon the braces between the end of the load and the abutment, when the partial load covers more than half the truss.

Eq. (249) may be put in this form:

$$H = \frac{wl}{4d} - \frac{wu}{2d}\left(1 - \frac{u}{2l}\right),$$

Under a full load, the least horizontal strain throughout the above distance, u, would be at the end of u; therefore, putting $x = u$, Eq. (244) becomes,

$$H = \frac{wl}{4d} - \frac{wu}{4d},$$

evidently greater than the strain from a partial load at the same place. Similarly, it may be shown that the longitudinal strain is greatest under a full load.

The above Eqs., (249) and (250), are true for any value of u, while x does not exceed $\frac{l}{2}$; that is, when the load covers less than half the truss, Eq. (249) is the horizontal, and Eq. (250) the longitudinal strain from the abutment to the centre, and being constant, do not cause any strain in the braces between those points.

Let Fig. (63) represent a truss less than half loaded.

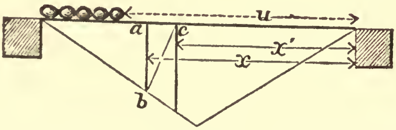


Fig. 63.

Let x represent the distance from the right abutment to a point between the centre of the truss and the end of the load, being therefore greater than $\frac{l}{2}$, but less than u; then,

$$\frac{l}{2} : d :: l - x : \frac{2d(l-x)}{l},$$

the vertical distance between the chords. And from moments around b,

$$\text{H}\,\frac{2d(l-x)}{l} = \frac{wx}{2l^2}(l-u)^2,$$

and

$$\text{H} = \frac{wx(l-u)^2}{4dl(l-x)}, \quad - \quad - \quad - \quad - \quad - \quad (251)$$

is the horizontal strain in the upper chord between the centre and the end of the load, and increases from the centre, where it is the same as Eq. (249), as x increases, to the end of the load, where $x = u$, and it becomes,

$$\text{H} = \frac{wu}{4dl}(l-u),$$

a strain very evidently, by comparison with Eq. (244), less than the strain at the same point under a full load.

This decrease in the value of H, as we pass from the centre, requires ties inclined as in full load, that is, as in Fig. (64), or struts with the opposite inclination. The less than half load requires the same braces as a full load.

Let

$$\text{H} = \frac{wx(l-u)^2}{4dl(l-x)},$$

be the compression at a, and

$$H' = \frac{wx'(l-u)^2}{4dl(l-x')}$$

be the compression at c, then

$$H' - H = \frac{w(l-u)(x-x')}{4d(l-x)(l-x')}$$

is the horizontal component of the strain in bc, and this component is to the vertical component as ac is to ab; or,

$$x - x' : \frac{2d(l-x)}{l} :: H - H' : \frac{w(l-u)^2}{2l(l-x')},$$

greatest when x' is greatest, or when it equals u, in which case we have,

$$V = \frac{w}{2l}(l-u), \quad \text{- - - - - } \quad (252)$$

for the vertical strain in the braces at the end of a less than half load.

The equation of the vertical strain at the same point from a full load is, Eq. (247),

$$V = \frac{wx}{2l};$$

but in this case x is the distance from the left abutment, or $l - u$; that is, the vertical strain in the braces from the less than half load is equal to the vertical strain at the same point from a full load.

Again, $\frac{w(l-u)^2}{2l^2}$, the reaction of the right abutment, is less than $\frac{w(l-u)}{2l}$; or the vertical strain in the braces at the end of the load is greater than the reaction of the unloaded abutment. This seeming contradiction, that the braces between the centre and the end of the load bear a greater weight than the abutment, can be readily explained.

In this case, and as H was determined,

$$L = \frac{wl'u}{2dl^2}(l - u),$$

the tension in the chord tie under the end of the load, and

$$l' : h :: \frac{wl'u}{2dl}(l - u) : \frac{wu}{2l^2}(l - u),$$

the vertical component of this longitudinal strain. This strain is passing towards the loaded end of the truss, or, in the present example, towards the left abutment. Subtracting this from the vertical strain in the brace, since it passes in the opposite direction,

$$\frac{w}{2l}(l - u) - \frac{wu}{2l^2}(l - u) = \frac{w}{2l^2}(l - u)^2,$$

the reaction of the right abutment, or the excess of vertical strain in the braces is neutralized by an excess of equal amount in the chord tie.

It is next necessary to ascertain the bracing required

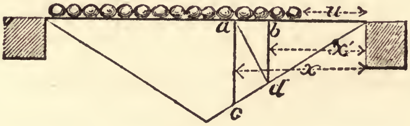


Fig. 64.

under a partial but greater than half load. In Fig. 64, let u equal the length of the unloaded part, as before, and x the distance from the abutment to a point under the load; then from moments around d we have,

$$\mathrm{H}\,\frac{2dx}{l} = \frac{wx}{2l^2}(l-u)^2 - \frac{w}{2l}(x-u)^2,$$

and

$$\mathrm{H} = \frac{w}{4dl}(l-u)^2 - \frac{w}{4dx}(x-u)^2, \quad - \quad - \quad (253)$$

in which H increases as x decreases; this increase requires ties inclined as in a full load, or as in Fig. (65); that is, as far as the centre, where H reaches its least value; beyond, H increases to the farther abutment, requiring ties with the opposite inclination.

At any point, b, from moments around c,

$$\mathrm{H}' = \frac{w}{4dl}(l-u)^2 - \frac{w}{4dx'}(x'-u)^2.$$

Whence,

$$\mathrm{H}' - \mathrm{H} = \frac{w}{4d}(x-x') - \frac{wu^2}{4d}\left(\frac{x-x'}{xx'}\right),$$

and, as before,

$$x - x' : \frac{2dx'}{l} :: \frac{w}{4d}(x-x') - \frac{wu^2}{4d}\left(\frac{x-x'}{xx'}\right) : \frac{wx}{2l} - \frac{wu^2}{2lx},$$

a strain passing in the same direction, and requiring the same kind of brace, but less by $\frac{wu^2}{2lx}$ than the vertical strain from a full load of equal density.

Hence we see that in every case the strains from the full load are the greatest, and that no braces are required for a partial load except those needed under a full load.

CASE I.—A SIMPLE TRIANGULAR TRUSS WITH VERTICAL STRUTS AND INCLINED TIES.

170. Example.—In Fig. 65,

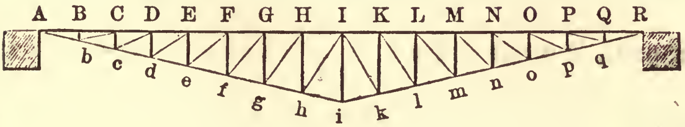


Fig. 65.

Let l = 200 feet, the length of the truss,
d = 20 feet, the depth of the truss at the centre,
p = 12.5 the length of a panel,
w = 200 tons, the weight supported by the truss when fully loaded.

The horizontal strains in the upper chord are obtained from Eq. (244), and the longitudinal strains in the lower-chord ties, from Eq. (241), whence the following table:

Values of x.	12.5	25	37.5	50	62.5	75	87.5	100
Strains in Tons, from Eq. (244).	375	350	325	300	275	250	225	
Compression in	AB, BC PQ & QR	CD & OP	DE & NO	EF & MN	FG & LM	GH & KL	HI & IK	
Strains in Tons, from Eq. (241).	386.6	360.9	335	309.3	283.5	257.8	231.9	206.2
Tension in	Ab & qR	bc & pq	cd & op	de & no	ef & mn	fg & lm	gh & kl	hi & ik

Eq. (247) gives the vertical strain in the ties, x being the distance to the abutment end of any tie; whence, from the proportion of the vertical to the longitudinal, we obtain the tensions in the ties.

The vertical strain in any strut distant x from the abutment, is a panel load, or $\frac{wp}{l}$, added to the vertical strain in the tie on the abutment side of the strut, or $\frac{wx'}{2l}$, when $x' = x - p$; substituting and adding, we have,

$$V = \frac{wp}{l} + \frac{w}{2l}(x - p) = \frac{wx}{2l} + \frac{wp}{2l}, \quad - \quad - \quad (254)$$

is the compression in any strut distant x from the abutment.

Whence the following table of strains in the braces:

Values of x.	12.5	25	37.5	50	62.5	75	87.5	100
Strains in Tons.	25.8	27.9	31.2	35.3	40.1	45.1	50.4	
Tension in	bc & Pq	cD & Op	dE & No	eF & Mn	fG & Lm	gH & Kl	hI & Ik	
Strains in Tons.	12.5	18.75	25	31.25	37.5	43.75	50	100
Compression in	Bb & Qq	Cc & Pp	Dd & Oo	Ee & Nn	Ff & Mm	Gg & Ll	Hh & Kk	Ii

The strain in Ii is double Eq. (254), as it receives the vertical strain from both sides.

CASE II.—A DOUBLE TRUSS WITH VERTICAL STRUTS AND INCLINED TIES.

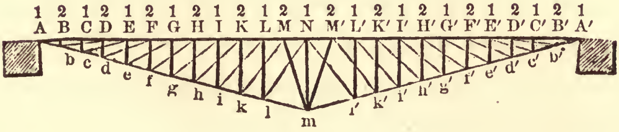


Fig. 66.

171.—This truss, like the preceding double trusses, may be divided into two simple trusses whose braces act independently of each other. The simple truss which has a full panel at the end will be designated as No. 1, the other as No. 2; their separate panel points are numbered in the figure. Each truss bears half the load.

Let l = the length of the truss,

d = the depth at the centre,

p = the length of a panel measured horizontally,

w = the weight, uniformly distributed.

172. Horizontal Strains.—Simple Truss No. 1 has the half-panel load resting directly upon the abutment, and has also a full panel at the end, hence the horizontal strain in the upper chord from this simple truss is

$$H = \frac{wl}{8d} - \frac{wx}{8d}. \qquad (255)$$

Simple Truss No. 2 has a full panel load a half (simple truss) panel length from the abutment, and its horizontal strain is similar to Eq. (162),

$$H' = \frac{wl}{8d} - \frac{wx'}{8d} + \frac{wp^2}{8dx'}. \qquad (256)$$

In the compound truss, the strain at any point in the upper or horizontal chord is the strain in one simple truss at the same point added to the strain in the other simple truss at the next panel towards the abutment. Hence, making x' of Eq. (256) $= x - p$, and adding to Eq. (255), we have for the horizontal chord strain at any panel point of Simple Truss No. 1,

$$H = \frac{wl}{4d} - \frac{wx}{4d} + \frac{wpx}{8d(x-p)}. \quad - \quad - \quad (257)$$

And making x of Eq. (255) $= x' - p$, and adding to Eq. (256), we have for the horizontal chord strain at any panel point of Simple Truss No. 2,

$$H = \frac{wl}{4d} - \frac{wx'}{4d} + \frac{wp}{8d} + \frac{wp^2}{8dx'}. \quad - \quad - \quad (258)$$

173. Vertical Strains in the Braces.—From Eq. (255) the vertical component of the strain in any tie of Simple Truss No. 1 is, where x is the horizontal distance to the lower end of the tie,

$$V = \frac{wx}{4l}. \quad - \quad - \quad - \quad - \quad - \quad - \quad (259)$$

The vertical strain in any strut is the vertical strain in that tie to which the upper end of the strut is attached, and one panel load, or $\frac{wp}{l}$; if x be the horizontal distance of the strut from the abutment, the strain in it is $\frac{wp}{l}$ + Eq. (259) at $x - 2p$, or

$$V = \frac{wp}{l} + \frac{w}{4l}(x - 2p) = \frac{wx}{4l} + \frac{wp}{2l}. \quad - \quad (260)$$

From Eq. (256) the vertical component of the strain in any tie of Simple Truss No. 2 is, where x' is the horizontal distance from the abutment to the lower end of the tie,

$$V = \frac{wx'}{4l} + \frac{wp^2}{4l(x' + 2p)}. \quad - \quad - \quad - \quad (261)$$

The vertical strain in any strut is, as before, the vertical strain in that tie to which the upper end of the strut is attached and one panel load; or when x' is the horizontal distance of the strut from the abutment, the compression in it is,

$$V = \frac{w}{4l}(x' - 2p) + \frac{wp^2}{4l(x' - 2p + 2p)} + \frac{wp}{l},$$

$$= \frac{wx'}{4l} + \frac{wp}{2l} + \frac{wp^2}{4lx'}. \quad - \quad - \quad - \quad (262)$$

174. Longitudinal Strains.—Taking moments around any panel point in the horizontal chord of Simple Truss No. 1, we obtain,

$$L = \frac{wl'}{4d} - \frac{wl'x}{4dl}. \quad - \quad - \quad - \quad - \quad - \quad (263)$$

And for Simple Truss No. 2,

$$L = \frac{wl'}{4d} - \frac{wl'x'}{4dl} + \frac{wl'p^2}{4dlx'}. \quad - \quad - \quad - \quad (264)$$

Making $x' = x + p$, and adding Eqs. (263) and (264), we have for the longitudinal strain in the chord ties of the compound truss at the panel points of Simple Truss No. 1,

$$L = \frac{wl'}{2d} - \frac{wl'x}{2dl} - \frac{wl'px}{4dl(x + p)}. \quad - \quad - \quad (265)$$

Next making $x = x' + p$, and adding Eqs. (263) and (264), we obtain, for the longitudinal strain in the compound truss at the panel points of Simple Truss No. 2,

$$L' = \frac{wl'}{2d} - \frac{wl'x'}{2dl} - \frac{wl'p}{4dl} + \frac{wl'p^2}{4dlxx'}. \quad \cdot \quad \cdot \quad (266)$$

175. Example.—Let Fig. (66) represent a triangular truss of the character described, in which

$l = 240$ feet, the length of the truss,

$l' = 123.69$ feet, the length of each section of the lower chord,

$d = 30$ feet, the depth of the truss at the centre,

$p = 10$ feet, the horizontal length of a panel,

$w = 300$ tons, the full uniform load.

The equations obtained above will apply to and give the strains in every member of the truss, except the two central horizontal chord-members. A part of the horizontal strain which the equation gives as belonging to them is taken by the inclined strut. The amount of horizontal strain in this strut may be obtained from its vertical strain by the proportion of the vertical to the horizontal extent of the strut. It is then to be deducted from the amount of strain obtained from the equation applicable to the horizontal member.

This change is owing to the uniformity of the truss (on which the equations were based) being broken by the inclination of the strut.

The following table gives the strains in the horizontal chord members :

Values of x in Eq. (257).	Values of x' in Eq. (258).	Strains in Tons.	Compression in
20		575.	AB, BC, CD & DC', CB', BA'
	30	541.67	DE & E'D'
40		516.67	EF & F'E'
	50	490.	FG & G'F'
60		465	GH & H'G'
	70	439.14	HI & I'H'
80		414.29	IK & K'I'
	90	388.89	KL & L'K'
100		363.89	LM & M'L'
	110	325.01	MN & NM'

The following table gives the tension in the lower-chord members:

Values of x' in Eq. (266).	Values of x' in Eq. (265).	Strains in Tons.	Tension in
10		592.65	Ab & b'A'
	20	558.26	bc & c'b'
30		532.46	cd & d'c'
	40	504.95	de & e'd'
50		479.15	ef & f'e'
	60	452.63	fg & g'f'
70		427.81	gh & h'g'
	80	400.60	hi & i'h'
90		374.80	ik & k'i'
	100	348.44	kl & l'k'
110		322.94	lm & m'l'

The following table gives the strains in the braces:

Values of x in Eq. (259).	20	40	60	80	100	
Strains in Tons.	25.78	27.95	29.47	35.35	40.02	
Tension in	cE & E′c′	eG & G′e′	gI & I′g′	iL & L′i′	lN & N′l′	
Values of x in Eq. (260).	20	40	60	80	100	120
Strains in Tons.	12.5	18.75	25	31.25	37.5	87.5
Compression in	Cc & C′c′	Ee & E′e′	Gg & G′g′	Ii & I′i′	Ll & L′l′	Nn & N′n′
Values of x in Eq. (261).	10	30	50	70	90	
Strains in Tons.	33.58	28.45	30.32	33.74	38.00	
Tension in	bD & D′b′	dF & F′d′	fH & H′f′	hK & K′h′	Km & M′k′	
Values of x in Eq. (262).	10	30	50	70	90	110
Strains in Tons.	12.5	16.67	22.5	28.57	34.61	43.12
Compression in	Bb & B′b′	Dd & D′d′	Ff & F′f′	Hh & H′h′	Kk & K′k′	Mn & M′n

The strain determined from the equations being multiplied by the secant of the angle of the brace to which it is applied.

CASE III.—A DOUBLE TRUSS WITH INCLINED STRUTS AND TIES.

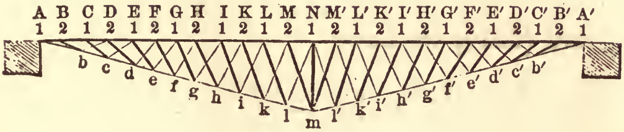


Fig. 67.

176.—This truss may be divided and numbered as in the last case.

Let l = the length of the truss,

l' = the length of each half of the lower chord,

d = the depth at the centre,

p = the horizontal length of a panel,

w = the maximum weight, uniformly distributed.

177. Horizontal Strains.—The moments of the load on any segment of the simple trusses are the same as in Case II., Chap. V. Hence, taking moments around any panel point of Simple Truss No. 1, in the lower chord, distant horizontally x from the nearest abutment, we have

$$\mathrm{H}\frac{2dx}{l} = \frac{wx}{4} - \frac{wx^2}{4l} - \frac{wp^2}{4l},$$

whence

$$\mathrm{H} = \frac{wl}{8d} - \frac{wx}{8d} - \frac{wp^2}{8dx}. \quad - \quad - \quad - \quad (267)$$

Similarly, from moments of Simple Truss No. 2, we have

$$\mathrm{H}' = \frac{wl}{8d} - \frac{w'x}{8d}. \quad - \quad - \quad - \quad - \quad (268)$$

The strain in the upper or horizontal chord on the abutment side of the point to which x is measured equals the strain at x, added to the strain at x', when $x' = x - p$; therefore, making $x' = x - p$, and adding Eqs. (268) and (267), we have

$$H = \frac{wl}{4d} - \frac{wx}{4d} + \frac{wp}{8d} - \frac{wp^2}{8dx}. \quad - \quad - \quad (269)$$

Again, making $x = x' - p$, and then adding the equations, we obtain for the horizontal strain in the members on the abutment side of the point to which x' is measured,

$$H' = \frac{wl}{4d} - \frac{wx'}{4d} + \frac{wp}{8d} - \frac{wp^2}{8d(x'-p)}. \quad - \quad (270)$$

178. Longitudinal Strains.—Taking moments around any panel point of No. 1, in the horizontal chord, we have

$$L\frac{dx}{l'} = \frac{wx}{4} - \frac{wx^2}{4l},$$

whence

$$L = \frac{wl'}{4d} - \frac{wl'x}{4dl}, \quad - \quad - \quad - \quad - \quad - \quad (271)$$

the tension in the lower chord.

And for No. 2,

$$L' = \frac{wl'}{4d} - \frac{wl'x'}{4dl} + \frac{wp^2l'}{4dlx'}. \quad - \quad - \quad - \quad (272)$$

Make $x' = x - p$, and adding, we have

$$L = \frac{wl'}{2d} - \frac{wl'x}{2dl} + \frac{wl'px}{4dl(x-p)}, \quad - \quad - \quad (273)$$

for the strain in any lower chord member whose centre end is opposite any upper-chord panel point of No. 1, distant x from the abutment.

Make $x = x' - p$, add, and we have

$$L = \frac{wl}{2d} - \frac{wl'x'}{2dl} + \frac{wl'p}{4dl} + \frac{wl'p^2}{4dlx'}, \quad - \quad - \quad (274)$$

for any lower-chord member whose centre end is opposite, or vertically beneath, a panel point of No. 2 distant x' from the abutment.

179. Vertical Strains.—Let Fig. (68) represent a segment of Simple Truss No. 1;

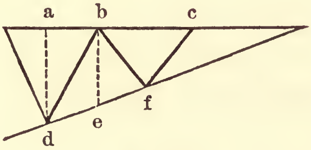


Fig. 68.

then from moments around d,

$$H = \frac{wl}{8d} - \frac{wx}{8d} + \frac{wp^2}{8dx}; \quad - \quad - \quad - \quad (275)$$

and from moments around e,

$$H' = \frac{wl}{8d} - \frac{wx'}{8d}. \quad - \quad - \quad - \quad - \quad (276)$$

Their difference,

$$H - H' = \frac{w}{8d}(x - x') + \frac{wp}{8dx'}$$

is the horizontal component of the compression in bd.

Its vertical component, from the proportion p or $x - x' : \frac{2dx}{l}$, is

$$V = \frac{wx}{4l} + \frac{wp}{4l}, \quad - \quad - \quad - \quad - \quad - \quad (277)$$

for the vertical component of the strain in the struts of No. 1, x being the horizontal distance to their lower ends.

From moments around e,

$$H = \frac{wl}{8d} - \frac{wx}{8d}. \quad - \quad - \quad - \quad - \quad - \quad (278)$$

From moments around f,

$$H' = \frac{wl}{8d} - \frac{wx'}{8d} - \frac{wp^2}{8dx'}. \quad - \quad - \quad - \quad (279)$$

Subtracting, we obtain, as before,

$$V = \frac{wx'}{4l} - \frac{wp}{4l},$$

for the vertical component of the strain in the ties of No. 1, x' being the horizontal distance to their lower ends.

For Simple Truss No. 2, by a similar process we obtain

$$V = \frac{wx}{4l} + \frac{wpx}{4l(x-p)}, \quad - \quad - \quad - \quad (280)$$

for the vertical component of the strain in the struts, x being the horizontal distance to their lower ends; and

$$V = \frac{wx'}{4l} - \frac{wpx'}{4l(x'+p)}, \quad - \quad - \quad - \quad (281)$$

for the vertical component of the strain in the ties, x' being the horizontal distance of their lower ends.

180. Example.—Let Fig. 67 represent a truss in which

$l = 240$ feet, the length of the truss,

$l' = 123.69$ feet, the length of each section of the lower chord,

$d = 30$ feet, the depth of the truss at the centre,

$p = 10$ feet, the horizontal extent of a panel,

$w = 300$ tons, the full uniform load.

Substituting these values in the above equations, we obtain the following table of strains for the different values of x and x':

Strains in Tons.	Compression in	Strains in Tons.	Tension in
575.	AB & A'B'	592.69	Ab & A'b'
550	BC & B'C'	558.32	bc & b'c'
533.33	CD & C'D'	532.46	cd & c'd'
508.33	DE & D'E'	505.06	de & d'e'
485	EF & E'F'	479.14	ef & e'f'
460	FG & F'G'	452.78	fg & f'g'
435.73	GH & G'H'	426.8	gh & g'h'
410.72	HI & H'I'	400.94	hi & h'i'
386.21	IK & I'K'	374.8	ik & i'k'
361.11	KL & K'L'	349.03	kl & k'l'
336.36	LM & L'M'	322.94	lm & l'm'
311.36	MN & M'N'		

The following is a table of the strains in the braces:

Strains in Tons.	Compression in	Strains in Tons.	Tension in
27.95	Bc & B'c'		
20.83	Cd & C'd'	9.32	Dc & D'c'
23.57	De & D'e'	10.41	Ed & E'd'
24.01	Ef & E'f'	14.14	Fe & F'e'
27.04	Fg & F'g'	16.01	Gf & G'f'
28.79	Gh & G'h'	19.32	Hg & H'g
31.94	Hi & H'i'	21.68	Ih & I'h'
34.19	Ik & I'k'	24.84	Ki & K'i'
36.90	Kl & K'l'	27.35	Lk & L'k'
39.90	Lm & L'm'	30.60	Ml & M'l'
43.12	Mn & M'n	33.25	Nm & Nm'
75	Nn		

CHAPTER IX.

BOW-STRING TRUSSES.

181.—A Bow-String Truss has one curved and one straight chord, the former meeting the latter at the abutments, and conveying to it the whole of its horizontal thrust, so that the reaction of the abutment is entirely vertical. The curved chord may be of any curvature, and on this depends to a great extent the amount of strain to which it will be subject. The chords are also connected by braces.

182. Parabolic Bow-String Trusses.—Let Fig. 69,

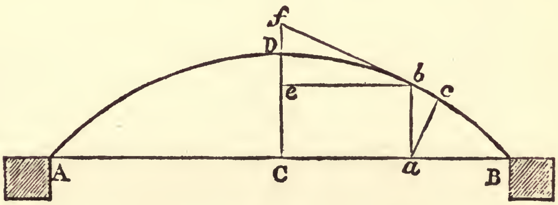


Fig. 69.

ADB represent the chords of a Parabolic Bow-String Truss, the braces being removed, whose arc, ADB, is the segment of a parabola, with the vertex at D. Let the chord AB receive the whole horizontal thrust of the arc, so that the abutments, A and B, receive only a downward vertical thrust from the truss and its load.

Let l = the length of the horizontal chord, AB,

d = the depth of the truss at the centre, DC, being always the versed sine of the arc,

w = the weight, uniformly distributed per unit of the horizontal chord, AB,

x = the horizontal distance of any point from one abutment,

y = the vertical distance between the chord and arc at x,

L = the longitudinal strain in the arc,

H = the horizontal strain in the chord,

V = the vertical strain or vertical component of a strain.

183. Longitudinal Strains.—Disregarding the braces, and taking moments around any point, a, distant x from the abutment B, in a vertical plane cutting no inclined braces, we have

$$\mathrm{L}(ac) = \frac{wx}{2} - \frac{wx^2}{2l} = \frac{wx(l-x)}{2l}, \quad - \quad - \quad (281)$$

$\frac{wx}{2}$ being the moment of the reaction of the abutment, and $\frac{wx^2}{2l}$ the moment of the load on x. The strain L at b is a thrust in the line of the tangent at that point; let bf be that tangent, then ac is its perpendicular distance from a, and $\mathrm{L} \times (ac)$ is the moment of the longitudinal strain at b.

Let D be the origin of the axes to which the parabola is referred, and we have from the equation of the parabola,

$$\frac{l^2}{4} = 2nd,$$

$$\text{or } 2n, \text{ the parameter, } = \frac{l^2}{4d}.$$

Again, from the equation of the parabola,

$$\left(\frac{l}{2} - x\right)^2 = \frac{l^2}{4d}(d - y),$$

$$\therefore \frac{l^2 y}{4d} = x(l - x), \quad - \quad - \quad - \quad - \quad (282)$$

and

$$y = \frac{4dx(l - x)}{l^2}. \quad - \quad - \quad - \quad - \quad (283)$$

The angle abc being equal to the angle bfe, in the similar triangles abc and bef,

$$fb : be :: ab : ac,$$

or let $t =$ tangent fb, and we have

$$t : \frac{l}{2} - x :: y : ac;$$

whence

$$ac = \frac{\left(\frac{l}{2} - x\right)y}{t}.$$

Substituting this, and the value of $x\ (l - x)$ of Eq (282) in Eq. (281), we have

$$\mathrm{L}\,\frac{\left(\frac{l}{2} - x\right)y}{t} = \frac{w}{2l}\left(\frac{l^2 y}{4d}\right);$$

whence,

$$L = \frac{wlt}{8d\left(\frac{l}{2} - x\right)}, \quad - \quad - \quad - \quad - \quad (284)$$

for the longitudinal strain in the arc at any point distant x horizontally from the abutment. Moving from any point towards the nearest abutment, t, the tangent, increases more rapidly than $\frac{l}{2} - x$, and consequently the longitudinal strain becomes the greatest at the abutment; moving from the abutment the strain decreases until we reach the centre, where $t = \frac{l}{2} - x$, when

$$L = \frac{wl}{8d}. \quad - \quad - \quad - \quad - \quad - \quad - \quad (285)$$

In the case of a truss of this form, the limits of x are 0 and l, since the uniformity of the truss is not broken as in the triangular truss.

184. Horizontal Strains.—Taking moments around the point b, we have for the strain in the horizontal chord,

$$Hy = \frac{wx}{2} - \frac{wx^2}{2l} = \frac{wx}{2l}(l - x).$$

Substituting the value of $x(l - x)$ in Eq. (282) we obtain

$$Hy = \frac{w}{2l}\left(\frac{l^2y}{4d}\right),$$

and

$$H = \frac{wl}{8d}, \quad - \quad - \quad - \quad - \quad - \quad (286)$$

a uniform strain throughout the chord and equal to the longitudinal strain in the arc at the centre; and since it is subject to no increase nor decrease throughout its length, there can be no horizontal strain in the braces, or no inclined braces are required under a uniform load. The load may rest upon the arc, or be suspended from it by ties, and still fulfil the conditions upon which the equations are based.

185.—In practice the arc is composed of a number of straight members, whose points of intersection are in the curve. In Fig. 70, let the arc be divided into a number

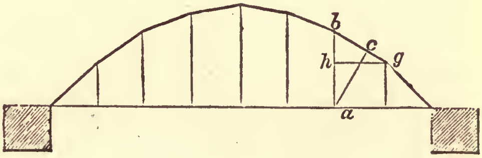


Fig. 70.

of short straight members by the division of the truss into an even number of panels of equal horizontal length, and let the load be considered as concentrated in equal amounts at the panel points. Then the moments of the abutment reaction, and of the load on the segment x remain the same; but L, the longitudinal strain, is now in the line of the arc member, instead of in the line of the tangent to the curve at the point x. The value of y, in Eq. (283), remains the same, because x is always measured to the panel ends, at which points the truss intersects the curve of the parabola.

In Fig. 70 let moments be taken around a; then, as before,

$$L \times (ac) = \frac{wx}{2l}(l - x),$$

and from similar triangles,

$$bg : gh :: y : ac.$$

Let $gh = p$, the horizontal length of a panel, and p' equal the length of the chord member bg; whence, substituting the value of y, Eq. (283), we have

$$p' : p :: \frac{4dx(l-x)}{l^2} : ac,$$

whence,

$$ac = \frac{4dpx(l-x)}{p'l^2},$$

then,

$$L\,\frac{4dpx(l-x)}{l^2p'} = \frac{wx(l-x)}{2l};$$

whence

$$L = \frac{wlp'}{8dp}, \quad - \quad - \quad - \quad - \quad - \qquad (287)$$

An equation in which we have but one variable p', and entirely independent of the values of x, and which shows that *the longitudinal strain in any arc member is to the horizontal chord strain as the length of that member is to its horizontal extent.*

The horizontal chord strain is not affected by the division of the truss into panels.

In Fig. (70) the truss is divided into an even number of panels, and d, the depth of the truss at the centre, is also the distance of the vertex of the parabola above the horizontal chord, or the versed size of the arc. If, however, the truss be divided into an odd number of panels, then d, in all the previous equations, is no longer the depth of the truss at the centre, because, at that point, the curve passes outside the truss. Hence it will be seen that d is the versed sine of the arc, and not always the depth of the truss.

186. The Horizontal Component of the Longitudinal or Arc Strain.—The horizontal component of the longi tudinal strain in any member is from the proportion

$$p' : p :: \frac{wlp'}{8dp} : \frac{wl}{8d},$$

a constant throughout the arc, and equal to the horizontal strain in the lower chord.

187. The Vertical Component of the Longitudinal Strain.—Let y be the vertical distance of the upper end of an arc member, and y' the vertical distance of the lower end of the same member above the lower chord. Then subtracting from the value of y, Eq. (283), at x, the value of y' at $x-p$, we have, after reduction,

$$y - y' = \frac{4dp(l - 2x + p)}{l^2}, \qquad (288)$$

the vertical extent of any arc member whose centre end is distant x from the nearest abutment, and from the proportion of its length to this vertical extent, we have

$$p' : \frac{4dp(l - 2x + p)}{l^2} :: \frac{wlp'}{8dp} : \mathrm{V} = \frac{w}{2} - \frac{wx}{l} + \frac{wp}{2l}, \qquad (289)$$

the vertical component of the strain in the arc member.

Whence it will be seen that the strain in each arc member is the resultant of all the vertical strain or weight that comes upon the truss between it and the centre, the whole weight concentrated upon its own centre end included, and the constant horizontal chord strain.

188. Strains from a Partial Load—Longitudinal Strains.—Let w' represent the weight of a full uniform load of equal density with the partial load, and let the load extend from one abutment a distance equal to $l - u$; (let the truss be divided as before into panels of equal horizontal length, and let u be measured to the centre of a panel). From the principles of the lever, $\frac{w'(l-u)^2}{2l^2}$ is the vertical reaction of the unloaded abutment; then, taking moments around a point in the horizontal chord in the unloaded part, x being less than u, we have,

$$L \times (ac) = \frac{w'x(l-u)^2}{2l^2}.$$

But

$$ac = \frac{4dpx(l-x)}{l^2p'},$$

$$\therefore L = \frac{w'p'(l-u)^2}{8dp(l-x)}, \quad - \quad - \quad - \quad - \quad (290)$$

an equation evidently greatest when x is greatest, or approaches u, or at the end of the load. If $l - x$ be made $= l - u$,

$$L = \frac{w'p'(l-u)}{8dp}, \quad - \quad - \quad - \quad (291)$$

less than Eq. (285), or at any point the longitudinal strain from a partial load is less than the strain from the full load.

189. Horizontal Strains from a Partial Load.—If moments be taken around any panel point in the arc in the unloaded part, x being less than u,

$$Hy = \frac{w'x(l-u)^2}{2l^2},$$

and by substituting value of y, Eq. (283),

$$H = \frac{w'(l-u)^2}{8d(l-x)}, \qquad (292)$$

which is also greatest at the end of the load, and, at any point, less than the strain from a full load.

If either this strain, or L, the longitudinal strain, be taken at any point under the load, it will be found to be less than the strain at the same point from a full load.

Eq. (292) varies with the different values of x, and consequently its changes must be taken or caused by inclined braces.

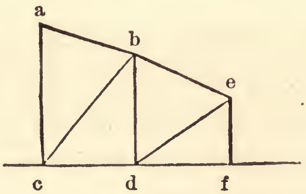


Fig. 71.

Let a b c d, Fig. (71), represent a panel in the unloaded part of a parabolic bow-string truss, and let x be the distance from the right or unloaded abutment to d, and x' the distance from the same abutment to c; then,

$$H = \frac{w'(l-u)^2}{8d(l-x)}$$

is the horizontal strain at d, and

$$H' = \frac{w'(l-u)^2}{8d(l-x')}$$

is the strain at c; now, since the horizontal chord is subject to tension, and the strain at c is greater than the strain at d, cb is a tie, and ac and bd struts.

190. Vertical Strains in the Braces from the Partial Load.—The horizontal component of the strain in bc is consequently $H' - H$, and its vertical component may be found from the proportion

$$cd : bd, \text{ or, } y : x' - x.$$

Subtracting $H' - H$, and putting for y its value, Eq. (283), we have,

$$x' - x : \frac{4dx(l-x)}{l^2} :: \frac{w'(l-u)^2(x'-x)}{8d(l-x')(l-x)} : V$$

$$= \frac{w'x(l-u)^2}{2l^2(l-x')}. \quad - \quad - \quad - \quad - \quad - \qquad (293)$$

In the case of a simple truss, or truss with but one system of braces, it was shown (54) that the reaction of the unloaded abutment does not equal the true amount of vertical strain in the brace at the end of the load, but that it is given by (Eq. 35),

$$V = \frac{w'n^2p^2}{2l(l-p)}.$$

In Fig. (71) x' is the horizontal distance from the abutment towards which the tie leans to the horizontal chord end of the tie, and x the horizontal distance from the same abutment to the arc end of the same tie. Therefore n, in Eq. (35), that is, the number of loaded panel points in $(l - u)$ of Eq. (293), is equal to $\frac{l - x - p}{p}$; and substituting $\frac{w'n^2p^2}{2l(l-p)}$ for $\frac{w'(l-u)^2}{2l^2}$ in Eq. (293), in place of n^2, its value just given, and for x', its value, $x + p$, we have,

$$V = \frac{w'(l - x - p)^2 x}{2l(l-p)(l-x-p)} = \frac{w'(l-x-p)x}{2l(l-p)}, \quad (294)$$

for the greatest vertical strain in any tie whose arc end is distant x, measured horizontally, from the abutment towards which the tie leans; which strain results solely from the moving load.

Under the effects of the moving load the vertical braces act as struts, and the greatest strain upon any one—bd, for example, in Fig. (71)—is when the panel point d is outside of the load, or when the tie cb is subject to its greatest tension. Then the strain in bd is equal to the vertical component of the strain in de, which may be determined from Eq. (293), x' and x being the distances to d and f. But (54) the panel point c cannot be fully loaded without a certain portion of the load coming upon the point d, which can cause no strain in bd; hence, as before, we must substitute $\frac{w'n^2p^2}{2l(l-p)}$ for $\frac{w'(l-u)^2}{2l^2}$, and x' being the distance of the vertical brace from the abutment, $n = \dfrac{l - x' - p}{p}$, $x = x' - p$, whence

$$V = \frac{w'x(l-u)^2}{2l^2(l-x')} = \frac{w'(l-x'-p)^2(x'-p)}{2l(l-p)(l-x')}, \quad - \quad - \quad (295)$$

is the compression in the vertical brace distant x' from the abutment, from the moving load w'.

From this is to be deducted the tension caused by the permanent load.

CASE I.—A SINGLE TRUSS, WITH VERTICAL AND INCLINED BRACES.

191. Example I.—Upper Chord Arched.—In Fig. (72),

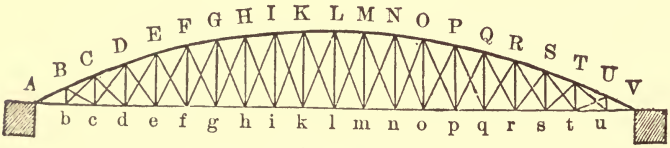


Fig. 72.

Let $l = 200$ feet, the length of the truss,

$d = 25$ feet, the depth of the truss at the centre,

$p = 10$ feet, the length of a panel,

$w' = 200$ tons, the weight of the full movable load,

$w = 100$ tons, the permanent truss weight.

Eq. (286), by substituting the values of the constants, becomes

$$H = \frac{(w' + w)l}{8d} = \frac{(200 + 100) \times 200}{200} = 300 \text{ tons},$$

the maximum tension throughout the length of the horizontal chord.

Similarly Eq. (285) becomes

$$L = 30p'.$$

Substituting the different values of p', or the lengths of the different arc-members, we can form the following table of strains in the upper or arched chord. The length of any arc-member is evidently the square root of the sum of the squares of the horizontal length of the panel and of the difference in height of the two ends of the arc-member.

Values of p'.	Strains in Tons.	Compression in
11.07	332.1	AB & UV
10.91	327.3	BC & TU
10.68	320.4	CD & ST
10.51	315.3	DE & RS
10.37	311.1	EF & QR
10.25	307.5	FG & PQ
10.15	304.5	GH & OP
10.07	302.1	HI & NO
10.03	300.9	IK & MN
10.00	300	KL & LM

When the truss is fully loaded, all the weight, except that of the arc, is borne by the vertical braces; if we assume that the weight of the arc, or the weight immediately upon it, is one-third of the permanent truss load, then, when the truss is fully loaded, $\frac{w'p}{l} + \frac{2wp}{3l}$ is the amount of tension upon each vertical brace. Substituting the values above, we have,

$$\frac{200 \times 10}{200} + \frac{2 \times 100 \times 10}{3 \times 200} = 13.33 \text{ tons},$$

for the maximum tension in each vertical brace.

The tension from the permanent truss load is $\frac{2wp}{3l}$, or 3.33 tons, which is to be deducted from the compression caused by the moving load. The vertical differ from the inclined braces in one respect: those of the latter, leaning in one direction, are affected only when the load is moving in the direction in which their arc ends are inclined; while the former are subject to strains from a load in either direction, and the greatest compression upon them is when the load covers more than half the truss; or when x', in Eq. (295), does not exceed $\frac{l}{2}$.

Substituting the values of the constants in Eq. (295) it becomes

$$V = \frac{(190 - x')^2(x' - 10)}{380(200 - x')},$$

whence, deducting the constant tension of 3.33 tons, we have the following table of maximum compressions in the vertical braces:

Values of x'.	Strains in Tons.	Compression in
20	1.19	Cc & Tt
30	4.60	Dd & Ss
40	7.77	Ee & Rr
50	10.42	Ff & Qq
60	12.55	Gg & Pp
70	14.16	Hh & Oo
80	15.22	Ii & Nn
90	15.83	Kk & Mm
100	15.85	Ll

There is no compression in Bb and Uu.

Substituting values in Eq. (294) it becomes

$$V = \frac{(90 - x)^2 x}{16000 - 160x};$$

whence, multiplying the vertical strains obtained therefrom by the secants of the tie angles, we have the following table of tensions in ties from the moving load

Values of x.	Strains in Tons.	Tension in
10	11.05	tU & Bc
20	13.37	sT & Cd
30	16.05	rS & De
40	18.62	qR & Ef
50	20.87	pQ & Fg
60	22.74	oP & Gh
70	24.15	nO & Hi
80	25.09	mN & Ik
90	25.54	lM & Kl
100	25.51	kL & Lm
110	24.97	iK & Mn
120	23.95	hI & No
130	22.43	gH & Op
140	20.40	fG & Pq
150	17.89	eF & Qr
160	14.89	dE & Rs
170	11.37	cD & St
180	7.08	bC & Tu

192. Example II.—The Lower Chord Arched—In Fig. (73),

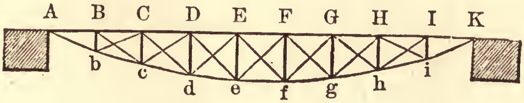


Fig. 73.

Let $l = 90$ feet, the length of the truss,

$d = 8.1$ feet, the versed sine of the arc,

$p = 10$ feet, the length of a panel,

$w' = 90$ tons, the weight of the full uniform movable load.

$w = 45$ tons, the weight of the permanent truss load.

The inversion of the arc results in many practical advantages; the longer chord becomes subject to tension, the shorter to compression, and the vertical braces to only one kind of strain, while the transverse bracing, to prevent flexure in the truss, does not interfere with the necessary headway.

For the compression in the horizontal chord we have Eq. (285),

$$H = \frac{wl}{8d} = 187.5 \text{ tons.}$$

For the tension in the arc we have Eq. (286),

$$L = 18.75p',$$

whence the following table for the different values of p':

Values of p'.	10	10.01	10.03	10.05
Strains in Tons.	187.5	187.8	188.1	188.4
Tension in	de, ef & fg	cd & gh	bc & hi	Ab & ik

It will be readily seen that Eq. (293) becomes in this case,

$$V = \frac{w'x'(l-u)^2}{2l^2(l-x)}, \qquad (296)$$

where x' is the distance, measured horizontally, from the abutment to the arc, or farthest end of the inclined tie. Making the change required, as explained in (54) for simple trusses, n of Eq. (35), the number of loaded panel points in $l-u$, is equal to $\frac{l-x'}{p}$, and $x=x'-p$, whence Eq. (296) becomes

$$V = \frac{w'(l-x')^2x'}{2l(l-p)(l-x'+p)}. \qquad (297)$$

Substituting the values given above, we have,

$$V = \frac{(90-x')^2x'}{16000-160\,x'}.$$

Multiplying the vertical strains obtained for the different values of x' by the secants of the angles, we form the following table of maximum tension in the ties.

Values of x.	20	30	40	50	60	70	80
Strains in Tons.	15.73	16.40	16.67	16.00	12.09	11.99	7.98
Tension in	hI & cB	gH & dC	fG & eD	eF & fE	dE & gF	cD & hG	bC & iH

Assuming, as in the last example, that one-third of the permanent truss load is borne directly by the arc, we have for the compression upon the vertical struts from the truss fully loaded, $\frac{w'p}{l}+\frac{2wp}{3l}=13.33$ tons.

The greatest compression upon these struts from the maximum moving load is when the ties to whose upper ends they are attached are subject to the greatest tension. This is given by Eq. (297), but the difference between this equation and Eq. (296) is also borne by the struts; hence Eq. (296) gives the total compression in the struts from the moving load. In this equation x is the distance from the abutment to the strut, $x'=x+p$, and $u=x+\frac{p}{2}$. Substituting these values, we have,

$$\mathrm{V}=\frac{w'(x+p)\left(l-x-\frac{p}{2}\right)^2}{2l^2(l-x)}=\frac{(x+10)(85-x)^2}{180(90-x)},$$

The greatest compression upon any strut is when x of this equation is less than $\frac{l}{2}$: that is, when the truss is more than half loaded. Adding 3.33 tons to the above, we have the following table:

Values of x.		20	30	40
Strains in Tons.	13.33	13.39	14.53	14.78
Compression in	Bb & Ii	Cc & Hh	Dd & Gg	Ee & Ff

The greatest compression in Bb and Ii is from the full load.

CASE II.—A COMPOUND TRUSS, WITH VERTICAL AND INCLINED BRACES.

193.—In Fig. (74),

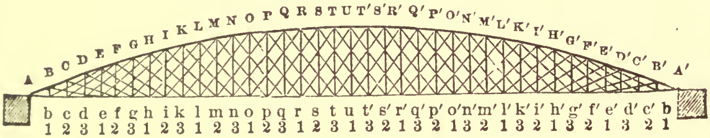


Fig. 74.

Let $l = 152$ metres, the length of the truss,

$d = 20$ metres, the depth of the truss at the centre,

$p = 4$ metres, the length of a panel, or distance between the vertical braces,

$w' = 500$ tons, the maximum movable load,

$w = 500$ tons, the permanent truss weight.

Under the full load this truss does not differ from the previous cases, and Eqs. (286) and (287) give the chord strains. Therefore,

$$\text{H} = \frac{wl}{8d} = \frac{1000 \times 152}{8 \times 20} = 950 \text{ tons},$$

is the tension throughout the horizontal chord; and

$$\text{L} = \frac{wlp'}{8dp} = 237.5p',$$

Multiplying by the different values of p', or lengths of the arc-members, we have the following table of compressions in the arc:

Values of p'.	Strains in Tons.	Compression in
4.49	1066.38	AB & B'A'
4.44	1054.5	BC & C'B'
4.39	1042.63	CD & D'C'
4.34	1030.75	DE & E'D'
4.30	1021.25	EF & E'F'
4.27	1014.13	FG & F'G'
4.23	1004.63	GH & G'H'
4.19	995.13	HI & I'H'
4.16	998	IK & K'I'
4.14	983.25	KL & L'K'
4.11	976.13	LM & M'L'
4.08	969	MN & N'M'
4.06	964.25	NO & O'N'
4.05	961.88	OP & P'O'
4.03	957.13	PQ & Q'P'
4.02	954.75	QR & R'Q'
4.01	952.38	RS & S'R'
4.00	950	ST & T'S'
4.00	950	TU & U'T'

Under the moving load we have three systems of braces acting independently of each other, as in the cases of the horizontal triple trusses. We will number these systems, or simple trusses, as Nos. 1, 2, and 3, their horizontal chord panel points being numbered in the figure. Eq. (293) gives the vertical component of the tensions in the inclined ties in either of these simple trusses when $\frac{w'(l-u)^2}{2l^2}$ of that equation representing the unloaded abutment reaction in a simple truss is changed to the amount of the abutment reaction upon each of these simple trusses.

In Simple Truss No. 1, which has a panel point next the abutment, the reaction of the farther abutment is $\frac{w'(l-u)^2 - \frac{p^2}{4}}{6l}$, as explained before. Eq. (293) therefore becomes

$$V = \frac{w'\left((l-u)^2 - \frac{p^2}{4}\right)x}{6l^2(l-x')}. \quad - \quad - \quad (298)$$

In this equation x is the distance to the arc end of the tie (from the abutment towards which the tie inclines), x' to the chord end, and equal to $x + 3p$, and u is equal to $x + \frac{3p}{2}$. Substituting these values, Eq. (298) may be reduced to this form:

$$V = \frac{w'x}{6l^2}\left(l - x + \frac{2p^2}{l - x - 3p}\right). \quad - \quad - \quad (299)$$

Inserting the constants in this equation we obtain for the different values of x the vertical components of the tensions in the inclined braces, which, multiplied by their secants, give the following table:

Values of x.	Strains in Tons.	Tension in
4	16.75	Be & B'e'
16	20.12	Eh & E'h'
28	20.88	Hl & H'l'
40	22.20	Lo & L'o'
52	25.15	Or & O'r'
64	26.39	Ru & R'u
76	26.84	Ur' & Ur
88	26.46	R'o' & Ro
100	25.47	O'l' & Ol
112	23.81	L'h' & Lh
124	22.36	H'e' & He
136	27.51	E'b' & Eb

If we assume that one-half of the permanent truss weight is borne directly by the arc, we shall have for the maximum tension upon each vertical brace, occurring when the truss is fully loaded,

$$\frac{w'p}{l} + \frac{wp}{2l} = 19.74 \text{ tons};$$

and for the tension from the permanent truss weight,

$$\frac{wp}{2l} = 6.58 \text{ tons}.$$

The greatest compression upon these braces is, as explained before, when they are outside the load extending from one abutment and covering the next panel point of the simple truss to which they belong.

This strain is given by Eq. (298), where, if x' be the distance from the unloaded abutment to any vertical brace, $x = x' - 3p$, and $u = x' + \frac{3p}{2}$. Substituting these values, Eq. (298) becomes,

$$V = \frac{w'\left\{\left(l - x' - \frac{3p}{2}\right)^2 - \frac{p^2}{4}\right\}(x' - 3p)}{2l^2(l - x')}. \qquad (300)$$

Subtracting the tension from the permanent weight from this equation, we obtain the table below of the maximum compressions. This equation is greater when x' is greater than $\frac{l}{2}$, or when the load covers less than half the truss, and for any value of x' greater than 124, or less than 28, the compression from the moving load is less than the constant tension.

Values of x.	124	112	100	88	76
Strains in Tons.	0.65	3.81	6.38	7.74	8.30
Compression in	Hh & H'h'	Ll & L'l'	Oo & O'o'	Rr & R'r'	Uu

Eq. (299) will also give the vertical strain in the inclined braces of Simple Truss No. 2. Simple Truss No. 1, in this example, is the same if the load enter at either end of the truss; the other simple trusses differ. As the equations depend upon the simple truss panel at that abutment from which the load extends, it will be seen that that equation which applies to the inclined braces from the points numbered 2 in the figure when the load is moving from the left to the right, also applies to the inclined braces from the points numbered 3 when the load moves in the opposite direction. Substituting values in Eq. (299), and multiplying by the secants of the tie angles, we have the following table:

Values of x.	Strains in Tons.	Tension in
132	24.34	Fc & F′c′
120	24.71	If & I′f′
108	24.46	Mi & M′i′
96	25.89	Pm & P′m′
84	26.71	Sp & S′p′
72	26.79	T′s & Ts′
60	25.02	Q′t′ & Qt
48	25.09	N′q′ & Nq
36	22.51	K′n′ & Kn
24	20.05	G′k′ & Gk
12	17.76	D′g′ & Dg

In Simple Truss No. 3, when the load is passing to the right, or that simple truss whose panel point is $3p$ from the abutment, Eq. (293) becomes,

$$V = \frac{w'\left((l-u)^2 - \frac{9p^2}{4}\right)x}{6l^2(l-x')}. \quad - \quad - \qquad (301)$$

As before, x is the distance from the abutment to the arc end of the tie, $u = x + \frac{3p}{2}$ and $x' = x + 3p$; whence Eq. (301) reduces to,

$$V = \frac{w'x(l-x)}{6l^2}. \quad - \quad - \quad - \quad - \qquad (302)$$

When the load is moving to the right this equation applies to the inclined braces from the points numbered 3, and to the inclined braces from the points numbered 2 when the load moves in the opposite direction.

Substituting values in Eq. (302), and multiplying by the secants of the tie angles, we have the following table:

Values of x.	Strains in Tons.	Tension in
128	20.01	Gd & G'd'
116	22.47	Kg & K'g'
104	23.58	Nk & N'k'
92	25.49	Qn & Q'n'
80	26.55	Tq & T'q'
68	26.50	S't & St'
56	25.57	P's' & Ps
44	24.01	M'p' & Mp
32	21.67	I'm' & Im
20	19.18	F'i' & Fi
8	16.83	C'f' & Cf

The vertical braces at the points numbered 2 are subject to different strains as the load moves to the right or left. The same is true of the vertical braces at the points numbered 3. The maximum compression upon one system of points, when the load is moving in one direction, is the same as that upon the other when it is moving in the opposite direction. If the load be moving to the right, the maximum compression is upon the verticals at the points 2, 2, etc.; and if to the left, upon the verticals at the points 3, 3, etc., by Eq. (300), from which we form the following table, the permanent load upon the horizontal chord being deducted.

Values of x.	36	48	60	72	84	96	108	120
Strains in Tons.	2.45	5.32	7.33	8.25	8.08	6.92	4.58	1.83
Compression in	Kk & K′k′	Nn & N′n′	Qq & Q′q′	Tt & T′t′	S′s′ & Ss	P′p′ & Pp	M′m′& Mm	I′i′ & Ii

The compression in Dd, Gg, G′g′, and D′d′ is less than the tension from the permanent load. The compression upon the verticals at the points 3, 3, etc., when the load moves to the right, or upon the verticals at 2, 2, etc., when to the left, may be determined from Eq. (302), as Eq. (300) was obtained from Eq. (298), and will be found to be less than that given above.

CASE III.—A SIMPLE TRUSS, WITH ALL THE BRACES INCLINED, AND HAVING EQUAL HORIZONTAL EXTENT.

194.—It is only when the intensity of the load is uniform horizontally upon the arc, or concentrated, or suspended by vertical ties in equal amounts, at points equidistant horizontally, that the strain in the horizontal chord is uniform throughout its length, and the strain in any arc-member is equal to the horizontal strain divided by the uniform panel length and multiplied by the length of that arc-member. This is evident when we consider that a brace can transmit only a longitudinal strain, and that consequently the strain in an inclined brace must have a horizontal as well as a vertical component.

The proportion of these components to each other depends on the inclination of the brace. If each brace in Fig. (75) be assumed to support a half panel load, then the horizontal component of its strain varies with the inclination of the brace, and the strain in the horizontal chord cannot remain uniform. If the horizontal components be equal in all the braces, then each brace cannot support the same amount of weight.

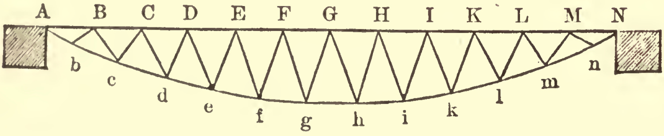


Fig. 75.

195.—Let Fig. (75) represent a Bow-String Truss, with a single system of inclined braces of uniform horizontal extent, the lower chord-points being in the curve of a parabola, as before;

Let l = the horizontal length of the truss,
d = the versed sine of the curve at the centre,
p = the length of the panels on the horizontal chord,
p' = the length of an arc-member,
w = the maximum load, permanent and movable,
x = the horizontal distance of any panel point from one abutment;
y = the vertical distance of any lower or arc chord panel point from the horizontal chord.

In the investigation of the chord strains w will be considered as concentrated at the horizontal chord panel points.

196. Horizontal Chord Strains.—Taking moments around any panel point in the arc, distant x from the abutment, we have,

$$\mathrm{H}y = \frac{wx}{2} - \frac{wx^2}{2l} - \frac{wp^2}{8l},$$

$\frac{wx^2}{2l} + \frac{wp^2}{8l}$ being the moment of the load on the segment x, and since $y = \frac{4dx(l-x)}{l^2}$, we have,

$$\mathrm{H} = \frac{wl}{8d} - \frac{wp^2 l}{32dx(l-x)}, \qquad (303)$$

for the strain in any horizontal chord-member whose centre is distant x from the abutment.

197. Longitudinal Arc Strains.—If moments be taken around any point in the horizontal chord, distant x from the abutment,

$$\mathrm{L}\left(\frac{y'+y}{2}\right)\frac{p}{p'}=\frac{wx}{2}-\frac{wx^2}{2l}, \qquad (304)$$

y' and y being the vertical distances of the two ends of the arc-member opposite the point x from the horizontal chord. Since y' is measured at $x+\frac{p}{2}$, and y at $x-\frac{p}{2}$,

$$\therefore \frac{y'+y}{2}=\frac{4d\left(x+\frac{p}{2}\right)\left(l-x-\frac{p}{2}\right)+4d\left(x-\frac{p}{2}\right)\left(l-x+\frac{p}{2}\right)}{2l^2},$$

$$=\frac{4d\left(lx-x^2-\frac{p^2}{4}\right)}{l^2}.$$

Substituting this in Eq. (304) we obtain,

$$\mathrm{L}=\left(\frac{wl}{8d}+\frac{wlp^2}{32d\left(lx-x^2-\frac{p^2}{4}\right)}\right)\frac{p'}{p}, \qquad (305)$$

for the strain in any arc-member whose centre is distant horizontally x from the abutment.

198. Vertical Strains in the Braces from a Full Load.—Let Eq. (303) represent the strain in any horizontal chord-member, and let

$$\mathrm{H}'=\frac{wl}{8d}-\frac{wp^2l}{32dx'(l-x')}.$$

represent the strain in the next member towards the centre of the truss, x' being equal to $x+p$, and confined to value less than $\frac{l}{2}$.

Since this strain increases from the abutment towards the centre, the difference must cause the same kind of strain in the braces whose arc ends are inclined towards that abutment from which x is measured, that there is in the horizontal chord; in this case compression, since this chord is uppermost.

The difference between the two strains is

$$\mathrm{H}' - \mathrm{H} = \frac{wlp^2[x'(l-x') - x(l-x)]}{32dx(l-x)x'(l-x')}. \qquad (306)$$

Its vertical component is, from the proportion,

$$\frac{p}{2} : \frac{4dx(l-x)}{l^2} :: \mathrm{H}' - \mathrm{H} : \mathrm{V}$$

$$= \frac{wp[x'(l-x') - (l-x)]}{4lx'(l-x')}, \quad - \quad - \quad - \qquad (307)$$

a quantity always less than $\frac{wl}{4p}$, or one-fourth of a panel load; hence the other brace is also subject to compression.

Had the vertical component of the difference in the horizontal strains been a panel load, it would prove the other brace to be subject to tension, or to no strain.

In Fig. 76, representing a segment from the right of the centre of a truss similar to Fig. (75), let the vertical section db be distant x from the right abutment, and the vertical section ca be distant x from the same abutment;

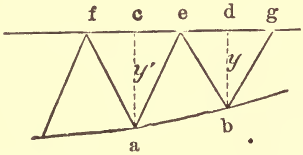


Fig. 76.

then Eq. (306) will represent the excess of horizontal strain in eb over that in ea, and Eq. (307) will represent the vertical strain, or the vertical component of the strain, of which Eq. (306) is the horizontal component. But this does not prove that there is a greater amount of vertical strain in eb than in ea, for they have different inclinations. From this vertical strain, however, may be determined the proportion of the panel load upon e, borne by ea or eb.

Deduct from $\frac{wp}{l}$, or the load upon the point e, the weight, Eq. (307), borne by the excess of horizontal strain in eb, and, of the remainder, let $V' =$ the amount borne by ea; and V, that borne by eb; then the horizontal components of the compressions in ea and eb, of which V' and V are the vertical components, are equal. Let H equal this horizontal component, uniting with V' or V, then

$$H : V' :: \frac{p}{2} : y', \text{ and } H : V :: \frac{p}{2} : y,$$

and

$$\frac{V'p}{2y'} = \frac{Vp}{2y}, \qquad \therefore V = \frac{V'y}{y'}, \quad - \quad - \quad - \quad (308)$$

and since $V'+V=\dfrac{wp}{l} - \dfrac{wp[x'(l-x')-x(l-x)]}{4lx'(l-x')}$.

Substituting the value of V, Eq. (308),

$$V'+V=V'\frac{(y'+y)}{y}.$$

Substituting the values of y' and y,

$$V'\frac{(y'+y)}{y} = V'\frac{x'(l-x')+x(l-x)}{x'(l-x')}.$$

$$\therefore V'\frac{x'(l-x')+x(l-x)}{'x(l-x')} = \frac{wp}{l} - \frac{wp[x'(l-x')-x(l-x)]}{4lx(l-x')}\Big\}$$

which may be reduced to,

$$V' = \frac{wp}{4l} + \frac{wpx'(l-x')}{2l[x'(l-x')+x(l-x)]}. \quad - \quad - \quad (309)$$

The vertical component of the strain in that brace whose arc end is farthest from the abutment from which x is measured, x' being the distance to the arc end and $x = x' - p$. Since, in Eq. (309), $\dfrac{x'(l-x')}{x'(l-x')+x(l-x)}$ is less than $\dfrac{1}{2}$, V' is greater than $\dfrac{wp}{2l}$, or, under a full load, the weight upon that brace whose arc end is nearer the centre is greater than the weight upon the other brace from the same horizontal chord point. This equation is necessary for the strain upon the braces from the permanent load.

The vertical strain upon the other brace from the same point in the horizontal chord is most readily determined by subtracting Eq. (309) from $\frac{wp}{l}$.

199. Vertical Strains in the Braces from a Movable Load.—In Fig. (76) let the load extend from the left abutment, so that the point e is not loaded. Then, since the chord can take no portion of the vertical strain, it follows that the vertical component of the strain in one of the braces, ae or be, is equal to that in the other, or V in ae = V in be; but since the inclinations of the braces differ, the horizontal components of their strains must differ also, and the sum of these horizontal components equals the difference in the horizontal strains in the chord-members fe and eg. If a be distant x', and b be distant x from the right and unloaded abutment, then $\frac{w'(l-u)^2}{2l^2}$ being the reaction of that abutment, w' being weight of the full movable load,

$$\mathrm{H}' = \frac{w'x'(l-u)^2}{2l^2y'}$$

is the strain in fe, and

$$\mathrm{H} = \frac{wx(l-u)^2}{2l^2y'}$$

the strain in eg.

The vertical components of the strains in ae and be are to their horizontal components as the vertical extents of these braces are to their horizontal extents, the latter being $\frac{p}{2}$; hence, $y' : \frac{p}{2} :: \mathrm{V} : \frac{\mathrm{V}p}{2y'}$, the horizontal component of the strain in ae, and $y : \frac{p}{2} :: \mathrm{V} : \frac{\mathrm{V}p}{2y}$, the horizontal component of the strain in be.

Therefore,

$$\frac{\mathrm{V}p}{2y'} + \frac{\mathrm{V}p}{2y} = \frac{\mathrm{V}p(y+y')}{2y'y} = \mathrm{H}' - \mathrm{H}$$
$$= \frac{w'x'(l-u)^2}{2l^2y'} - \frac{wx(l-u)^2}{2l^2y},$$

and

$$\mathrm{V}p(y+y') = \frac{w'(l-u)^2}{l^2}(x'y - xy')$$

Substitute for y' and y their values as given in Eq. (283), and we have, omitting $\frac{4d}{l^2}$, since it is common to all the members of the equation,

$$\mathrm{V}p[x(l-x)+x'(l-x')] = \frac{w'(l-u)^2[x'x(l-x) - xx'(lx-')\}}{l^2}$$
$$= \frac{w'(l-u)^2xx'(x'-x),}{l^2};$$

and since $p = x' - x$

$$\therefore \quad \mathrm{V} = \frac{w'(l-u)^2xx'}{l^2[x(l-x)+x'(l-x')]}, \quad - \quad - \quad (310)$$

is the vertical component of the strain in either ae or be, and is greatest when $(l-u)^2$ is greatest, or when the load extends from the abutment opposite to that from which x' x, and u are measured, and fully loads the panel point next these two braces; in this case the panel point f.

The longitudinal strains in ab and be have a resultant at b, which passes in the direction of the end of the truss resting on the (in this case right) abutment. For let y, the vertical depth at b, represent the vertical abutment reaction, then V : H :: $y : x$; or,

$$\frac{w(l-u)^2}{2l^2} : \frac{w(l-u)^2}{8d(l-x)} :: \frac{4dx(l-x)}{l^2} : x.$$

Therefore the amount only, and not the direction of this resultant, is affected by the weight of the load. The relative proportions of the components of this strain borne by ab and be consequently remain the same, even when the point e is loaded. Hence Eq. (310) will determine the strain in fa when f is loaded, as well as in ae and be; and the greatest compression in fa, and tension in ae, are when the point f is loaded.

To apply the equation, since it is a single truss, $\frac{w'n^2p^2}{2l(l-p)}$ must be substituted (54) for $\frac{w'(l-u)^2}{2l^2}$; and since $x' = x + p$, $n = \frac{l - x - \frac{p}{2}}{p}$, Eq. (310) becomes =

$$V = \frac{w'\left(l - x - \frac{p}{2}\right)^2 x(x+p)}{l(l-p)[2x(l-p-x)+p(l-p)]}, \quad (311)$$

for the vertical component of the

greatest $\left\{\begin{array}{l}\textit{tension} \text{ when the arc is above,}\\ \textit{compression} \text{ when the arc is below,}\end{array}\right\}$ from a movable load in any brace whose arc and nearest end is distant x from the abutment. To this strain must be added the strain from the permanent load, which is $\frac{wp}{l}$ — Eq. (309), or,

$$V = \frac{3wp}{4l} - \frac{wpx''(l-x')}{2l\,[x'(l-x') + x(l-x)]},$$

x' being equal to $x+p$, this becomes

$$V = \frac{wp}{4l} + \frac{wpx(l-x)}{2l[2x(l-p-x)+p(l-p)]}. \quad (312)$$

In this equation w represents the weight of the permanent truss load, and this equation, added to Eq. (311), gives the total maximum vertical strain in those braces to which Eq. (311) applies.

Making $x = x' - p$, $n = \dfrac{l - x' - \frac{p}{2}}{p}$, Eq. (310), becomes

$$V = \frac{w'\left(l - x' - \frac{p}{2}\right)^2 x'(x'-p)}{l(l-p)[2x'(l+p-x') - p(l+p)]}, \quad (313)$$

for the vertical component of the greatest $\left\{\begin{matrix}\text{compression}\\ \text{tension}\end{matrix}\right\}$ when the arc is $\left\{\begin{matrix}\text{above}\\ \text{below}\end{matrix}\right\}$, from a movable load in any brace whose arc and farthest end is distant x' from the abutment. From this must be subtracted Eq. (309), giving the strain from the permanent load in which x is to be made equal to $x' - p$.

200. Example.—In Fig. (75),

Let $l = 120$ feet, the length of the truss,

$d = 14.4$ feet, the versed sine of the curve of the lower chord,

$p = 10$ feet, the horizontal distance between the panel points,

$w' = 120$ tons, the weight of the full movable load,

$w = 48$ tons, the permanent truss weight.

Substituting values in Eq. (303), w being changed to $w' + w$, we can form the following table of strains in the horizontal chord:

Values of x.	5	15	25	35	45	55
Strains in Tons.	167.4	172.2	173.2	173.5	173.7	173.8
Compression in	AB & MN	BC & LM	CD & KL	DE & IK	EF & HI	FG & GH

For the strains in the arc, we obtain from Eq. (305), w being changed to $w' + w$, the following table:

Values of x.		10	20	30	40	50	60
Values of p'.		10.77	10.50	10.28	10.03	10.02	10
Strains in Tons.	184.1	192.9	186.1	181.6	176.9	176.6	176.2
Tension in	Ab & nN	bc & mn	cd & lm	de & kl	ef & ik	fg & hi	gh

The strain in the first, or end member, is found from the strain in the horizontal end member, dividing by 5 its horizontal extent, and multiplying by 5.5 its length.

Supposing one-half of the permanent truss weight to be borne directly by the arc, w of Eq. (309) is equal to 24 tons; we have from Eqs. (311) and (312) the following table of compression in the braces, the vertical strain being multiplied by the secant of the brace angle.

Values of x.	Strains in Tons.	Compression in
5	4.60	Mn & Bb
15	9.52	Lm & Cc
25	12.99	Kl & Dd
35	15.40	Ik & Ee
45	16.85	Hi & Ff
55	17.36	Gh & Gg
65	16.96	Fg & Hh
75	16.63	Ef & Ii
85	13.41	De & Kk
95	10.28	Cd & Ll
105	6.34	Bc & Mm

Similarly from Eqs. (313) and (309), we obtain the following table of tensions in the braces ;

Values of x.	Strains in Tons.	Tension in
15	2.97	Mm & Bc
25	6.78	Ll & Cd
35	9.29	Kk & De
45	10.75	Ii & Ef
55	11.28	Hh & Fg
65	10.99	Gg & Gh
75	9.91	Ff & Hi
85	7.93	Ee & Ik
95	5.25	Dd & Kl
105	2.15	Cc & Lm

CASE IV.—A DOUBLE TRUSS, WITH ALL THE BRACES INCLINED, AND HAVING EQUAL HORIZONTAL EXTENT.

201.—Let Fig. (77) represent a bow-string parabolic Truss, containing two systems of braces, which permit

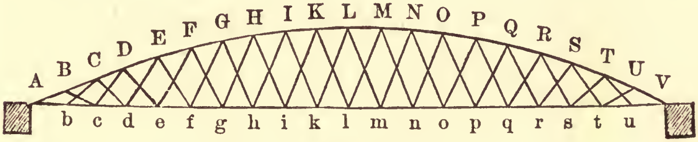


Fig. 77.

the division of the truss into two simple trusses, whose chords are common, but whose braces act independently of each. Let Fig. (78) represent one of these trusses, designated as Simple Truss No. 1, and Fig. (79) the other, designated as Simple Truss No. 2.

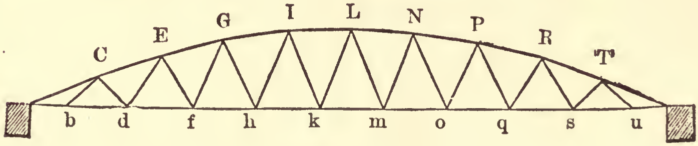


Fig. 78.

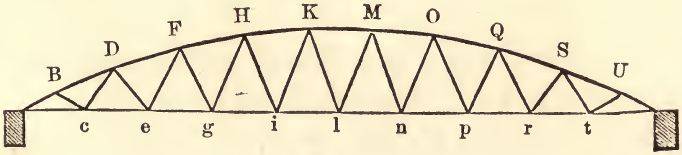


Fig. 79.

202.—Let $l =$ the horizontal length of the truss,

$d =$ the versed sine of the curve,

p = the horizontal length of the panels of the double truss,

p' = the length of an arc member,

w = the maximum load, uniformly distributed,

x = the horizontal distance of any panel point from one abutment,

$y = \frac{4dx(l-x)}{l^2}$, the vertical distance between the chords at any panel point x.

203. Horizontal Chord Strains.—Each truss bears one-half the load, and the half panel load at each abutment is taken as belonging to No. 2. Taking moments around a point in the arc chord of No. 1. Fig. (80), distant x from the abutment,

$$\mathrm{H}\frac{4dx(l-x)}{l^2} = \frac{wx}{4} - \frac{wx^2}{4l},$$

$\therefore$

$$\mathrm{H} = \frac{wl}{16d}, \qquad (314)$$

is the tension in the lower, or horizontal chord, between the points b and u. But outside of these points, or between them and the abutment, this equation will not apply, because there is no panel point in the upper chord of this truss. The manner of determining the strain in the end member will be shown hereafter.

Taking moments around the upper chord panel points of No. 2, we have,

$$H' \frac{4dx(l-x)}{l^2} = \frac{wx}{4} - \frac{wx^2}{4l} - \frac{wp^2}{4l},$$

whence,

$$H' = \frac{wl}{16d} - \frac{wp^2 l}{16dx(l-x)}, \quad - \quad - \quad - \quad (315)$$

is the tension throughout the horizontal chord of No. 2.

Adding these equations, (314) and (315), we have,

$$H + H' = \frac{wl}{8d} - \frac{wlp^2}{16dx(l-x)}, \quad - \quad - \quad - \quad (316)$$

for the strain in the horizontal chord of the double truss, Fig. (77) in which x is the horizontal distance to the upper chord points of Truss No. 2, which is the same as to the lower chord points of No. 1, and gives the strains in the members on both sides of these points. That is, since the strain in No. 1 is constant, the strain in the double truss only changes as we pass the points of No. 2.

For the horizontal strain in the end member take moments about T or C, the first upper chord panel points of Simple Truss No. 1, and we have

$$H \frac{4dx(l-x)}{l^2} = \frac{w}{4} \times 2p.$$

The moment of the load on the first lower chord panel point of this simple truss at u or b does not affect the equation for the strain in the member on the abutment side of either of these points. Since $x = 2p$, this equation becomes,

$$H = \frac{wl^2}{16d(l-2p)}. \quad - \quad - \quad - \quad - \quad - \quad (317)$$

Add this to Eq. (315), which extends throughout the horizontal chord, and making $x = p$, we have,

$$H = \frac{wl}{16d} - \frac{wpl}{16d(l-p)} + \frac{wl^2}{16d(l-2p)}, \quad (318)$$

for the strain in the end member of the horizontal chord.

204. Longitudinal Arc Strains.—Taking moments around the horizontal chord points of Simple Truss No. 1, we obtain

$$L\frac{p}{p'}\frac{4dx(l-x)}{l^2} = \frac{wx}{4} - \frac{wx^2}{4l} + \frac{wp^2}{4l},$$

whence,

$$L = \left(\frac{wl}{16d} + \frac{wlp^2}{16dx(l-x)}\right)\frac{p'}{p}, \quad - \quad - \quad - \quad (319)$$

is the compression in all the arc members of Simple Truss No. 1, except the member at either end; to which, from the manner of taking the moments, the equation is not applicable.

Taking moments around the horizontal chord panel points of Simple Truss No. 2, we obtain,

$$L\frac{p}{p'}\frac{4dx(l-x)}{l^2} = \frac{wx}{4} - \frac{wx^2}{4l},$$

whence,

$$L = \frac{wlp'}{16dp}, \quad - \quad - \quad - \quad - \quad - \quad (320)$$

is the strain in all, except, as before, the end members of the arc of No. 2.

Adding Eqs. (319) and (320) we have

$$L = \left(\frac{wl}{8d} + \frac{wlp^2}{16dx(l-x)}\right)\frac{p'}{p}. \quad - \quad - \quad - \quad (321)$$

The compression in the upper chord members of the double truss: here x is the distance to the lower or horizontal chord panel points of No. 1, and for each value of x there are two values of p', the lengths of the two members meeting at that point.

The compression in the end arc member has a horizontal component equal to the strain in the horizontal end member, or Eq. (318) $\times \frac{p'}{p}$, or

$$L = \left(\frac{wl}{16d} - \frac{wlp}{16d(l-p)} + \frac{wl^2}{16d(l-2p)}\right)\frac{p'}{p}, \quad (322)$$

is the strain in the end member.

The horizontal component of the strain in the end members of No. 1, that is, from A to C and from T to V, is given by Eq. (317), and in the second members of Truss No. 2 is given by Eq. (320); adding these equations, and multiplying by $\frac{p'}{p}$, we obtain

$$L = \left(\frac{wl}{16d} + \frac{wl^2}{16l(l-p)}\right)\frac{p'}{p}. \quad - \quad - \quad - \quad (323)$$

205. Vertical Strains in the Braces from a Full Load.—In Simple Truss No. 1 it has been shown that the strain in the horizontal chord is, with the exception of the member at either end, uniform throughout that chord. The horizontal components of the strains in any two braces of this simple truss, meeting at the same horizontal chord point, are therefore equal. Let $h =$, the horizontal component of the strain in either of the braces meeting at o in Fig. (78); let $v =$, the vertical component of the strain in the brace to the point P, and $v' =$, the vertical component of the strain in the brace to the point N; then these two vertical components, $v' + v = \frac{wl}{p}$, or the weight upon the point o.

Since

$$p : y :: h : v \therefore h = \frac{pv}{y},$$

and

$$p : y' :: h : v' \therefore h = \frac{pv'}{y'}.$$

Then $\frac{v'}{y'} = \frac{v}{y}$, and $v = \frac{v'y}{y'}$.

$\therefore$

$$v' + \frac{v'y}{y'} = \frac{wp}{l},$$

and

$$v' = \frac{wpy'}{l(y' + y)}.$$

Substituting for y' and y their values, $\frac{4dx'(l - x')}{l^2}$ and $\frac{4dx(l - x)}{l^2}$, we have,

$$V = \frac{wpx'(l - x')}{l(x'(l - x') + x(l - x))} \quad - \quad - \quad (324)$$

for the vertical component of the strain in that brace whose arc end leans from the abutment from which x' is measured, x being $= x' - 2p$. The vertical component of the strain in the other brace from the same point is $\frac{wp}{l}$ — Eq. (324). It will be observed that, since

$x'(l-x')+x(l-x)$ is less than $2x'(l-x')$ when x' is not greater than $\frac{l}{2}$; Eq. (324) is greater than $\frac{wp}{2l}$, or, the brace leaning towards the centre supports more than half a panel load.

In Simple Truss No. 2 we have the same case which was explained in (193), and to which, consequently, Eq. (309) will apply.

206. Vertical Strains in the Braces from a Moving Load.—A comparison of these Simple Trusses with Case (III) will show that the vertical strains, or vertical components of the strains in the braces, are similar; and that Eq. (310) representing twice the reaction of the abutment from which x is measured, multiplied by $\frac{l'x}{x'(l-x')+x(l-x)}$, ($x$ being the distance from the abutment whose reaction is taken to the arc end of any brace leaning towards the same abutment, x' being $=x+2p$, and u being $=x$,) will give the maximum tension in that brace.

Hence, in Simple Truss No. 1, the reaction of the abutment from a partial load, found as in the horizontal trusses, being, $\frac{w'(l-u)^2}{4l^2}$, Eq. (310) becomes,

$$V=\frac{w'(l-x)^2x(x+2p)}{2l^2[x(l-x)+(+2p)(l-xx-2p)]},$$

$$=\frac{w'(l-x)^2x(x+2p)}{4l^2[x(l-2p-x)+p(l-2p)]}. \quad - \quad - \quad (325)$$

This being tension, the tension from the constant truss weight is to be added to it.

The greatest compression in the braces of the same simple truss is, as has been explained in the previous case, given by Eq. (310), u being $= x'$ and $x = x' - 2p$, whence Eq. (310) becomes

$$V = \frac{w'(l - x')^2 x'(x' - 2p)}{2l^2[(x' - 2p)(l - x' + 2p) + x'(l - x')]}$$

$$= \frac{w'(l - x')^2 x'(x' - 2p)}{4l^2[x'(l + 2p - x) - p(l + 2p)]}, \quad (326)$$

x' being the horizontal distance from the unloaded abutment to the arc end of the brace leaning from that abutment; and since the strain is compression, it is necessary to deduct from it the strain from the constant load.

In Simple Truss No. 2, by a similar process of reasoning, Eq. (310) becomes changed to

$$V = \frac{w'[(l - x)^2 - p^2]x(x + 2p)}{4l^2[x(l - x - 2p) + p(l - 2p)]}, \quad (327)$$

for the tension in those braces whose arc ends are distant x horizontally from the abutment towards which they lean.

And for the compression in those braces whose arc ends are distant x' horizontally from the abutment from which they lean, Eq. (310) becomes changed to

$$V = \frac{w'[(l - x')^2 - p^2]x'(x' - 2p)}{4l^2[x'(l + 2p - x) - p(l + 2p)]}, \quad (328)$$

To Eq. (327) is to be added the constant truss load, Eq. (324); and from Eq. (328) is to be subtracted the constant truss load to attain the maximum result.

207. Example.—In Fig. (77),

Let $l = 200$ feet, the length of the truss,

$d = 25$ feet, the depth of the truss at the centre, and the versed sine of the curve,

$p = 10$ feet, the horizontal distance between the panel points,

$w = 100$ tons, the permanent truss weight, two-thirds of which is supported by the braces,

$w' = 200$ tons, the weight of the full movable load.

From Eq. (318) for the end member, and Eq. (316) for the other members, we obtain the following table of strains in the horizontal chord, w of these equations being the maximum load, 300 tons:

Values of x.		10	30	50	70	90
Strains in Tons.	308.77	292.11	297.06	298.00	298.35	298.49
Tension in	Ab & uV	bc & tu	cd, de, rs, & st	ef, fg, pq. & qr	gh, hi, no & op	ik, kl, lm & mn

From Eqs. (321), (322), and (323), we may form the following table of compressions in the arc chord; w being here also the maximum load, 300 tons:

Values of x.	Strains in Tons.	Compression in
	341.81	AB & UV
	334.37	BC & TU
30	323.54	CD & ST
30	318.39	DE & RS
50	313.17	EF & QR
50	309.55	FG & PQ
70	306.17	GH & OP
70	303.76	HI & NO
90	302.41	IK & MN
90	301.81	KL & LM

From Eq. [325), added to two-thirds of $\frac{wp}{l}$ — Eq. (324), we obtain the tensions; and from Eq. (326), less two-thirds of Eq. (324), the compressions in the braces of Simple Truss No. 1, as given in the following table, the vertical strains being multiplied by the secants of the angles of the braces.

Values of x & x'.	20	40	60	80	100	120	140	160	180
Strains in Tons.	11.48	13.94	16.18	16.60	18.32	18.09	17.18	16.07	19.92
Tension in	Cd & Ts	Ef & Rq	Gh & Po	Ik & Nm	Lm & Lk	No & Ih	Pq & Gf	Rs & Ed	Tu & Cb
Strains in Tons.		3.53	6.71	8.48	9.17	8.85	7.58	5.44	2.51
Compression in		Rs & Ed	Pq & Gf	No & Ih	Lm & Lk	Ik & Nm	Gh & Po	Ef & Rq	Cd & Ts

Again, from Eq. (327), added to two-thirds of $\frac{wp}{l}$ — Eq. (309), we obtain the tensions, and from Eq. (328), less two-thirds of Eq. (309), the compressions in the braces of Simple Truss No. 2, as given in the following table; the vertical strain being multiplied by the secants of the angles of the braces:

Values of x & x'.	10	30	50	70	90	110	130	150	170
Strains in Tons.	12.40	12.81	15.18	17.00	17.97	18.06	17.79	16.41	15.80
Tension in	Bc & Ut	De & Sr	Fg & Qp	Hi & On	Kl & Ml	Mn & Ki	Op & Hg	Qr & Fe	St & Dc
Strains in Tons.		1.84	5.47	7.76	8.89	8.98	8.52	6.15	3.24
Compression in		St & Dc	Qr & Fe	Op & Hg	Mn & Ki	Kl & Ml	Hi & On	Fg & Qp	De & Sr

208.—By similar processes equations may be prepared for Parabolic Bow String Trusses containing any number of Simple Trusses, or single systems of bracing.

209. Bow String Trusses, with Arcs of any Curvature.—It is only when y, or the vertical distance between the arc and the horizontal chord at any point, bears some ratio to the horizontal distance of that point from the abutment, that we can express the former in terms of the latter, as in the Parabolic Bow String. This can be done when the curve can be referred to rectangular axes, as is the case with the circle, ellipse, and many others. But in these cases the equations become more intricate, and it is better that they should contain two variables than to be rendered liable to error by their prolixity.

The following equations will apply to any Bow String Truss, as they are entirely independent of the curvature of the arc.

210.—Let l = the length of the truss,

p = the horizontal length of a panel.

p' = the length of any arc member,

w = the weight, maximum and uniform, upon the truss,

x = the horizontal distance of any panel point from one abutment,

y = the vertical distance between the chords at x

211. Horizontal Strains.—From moments around any panel point in the arc of Fig. (80), we

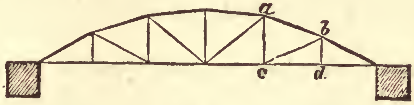


Fig. 80.

have for the tension in the horizontal chord,

$$Hy = \frac{wx}{2} - \frac{wx}{2l},$$

whence,

$$H = \frac{wx}{2y} - \frac{wx^2}{2ly}, \quad - \quad - \quad - \quad - \quad (329)$$

and at any other point x',

$$H' = \frac{wx^2}{2y'} - \frac{wx'^2}{2ly'}. \quad - \quad - \quad - \quad (330)$$

If H' be greater than H, then the horizontal chord end of the inclined brace between the two points x' and x must be distant x', x' and the arc end distant x from the abutment. That is, if the horizontal strain at c be greater than the horizontal strain at d, a brace is necessary from c to b. If, on the contrary, the strain be greater at d, then the brace must be from a to d. The vertical components of the strains in these braces, necessary to ascertain the effects of the constant load, may be obtained, as has been shown before, from the difference in the horizontal strains.

212. Longitudinal Arc Strains. — From moments around any point in the horizontal chord we have,

$$L = \frac{p'}{p}\left(\frac{wx}{2y} - \frac{wx'^2}{2ly}\right). \quad - \quad - \quad - \quad (331)$$

Eqs. (329) and (331) apply to the members of the chords on that side of any point to which x is measured, on which, under a full load, no brace is in action.

213. Vertical Strains in the Braces from a Moving Load.—Under a partial load, extending from one abutment a distance $l-u$, at any point, x, outside the load, x and u being measured from the same abutment, and w' being the weight of a full load of equal density with the partial load, the strain in the lower chord will be,

$$H = \frac{w'(l-u)^2x}{2l^2y}; \quad - \quad - \quad - \quad - \qquad (332)$$

and at the next panel point x',

$$H' = \frac{w'(l-u)^2x'}{2l^2y'}. \quad - \quad - \quad - \quad - \qquad (333)$$

Subtracting

$$H - H' = \frac{w'(l-u)^2(xy' - x'y)}{2l^2yy'}. \quad - \qquad (334)$$

Let $x' = x - p$, then

$$H - H' = \frac{w'(l-u)^2(xy' - xy + py)}{2l^2yy'}, \qquad (335)$$

is the difference in the horizontal strains at two consecutive panel points outside, and consequently the horizontal component of the strain in the brace. If y' be the vertical extent of this brace between x and $x-p$, p being its horizontal extent, we have from the proportion

$$p : y' :: H - H' : V = \frac{w'(l-u)^2[x(y'-y)+py]}{2l^2py}, \qquad (336)$$

for the vertical component of the strain in any inclined brace between the panel points x and $x-p$.

This equation must be changed, as has been done in the previous cases to suit different systems of bracing; $\frac{w'(l-u)^2}{2l^2}$ be altered to represent the reaction of the unloaded abutment of the simple truss to which it is to be applied, and p is always to represent the horizontal length of a simple truss panel. Examples of the application of these equations are considered unnecessary.

CHAPTER X.

LENTICULAR TRUSSES.

214.—The form of this peculiar truss, known also as the Pauli System, is shown in the following figure:

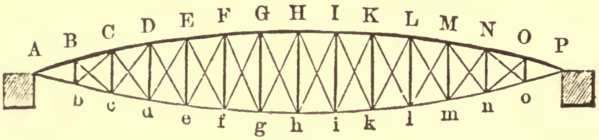


Fig. 81.

It is composed of two equal parabolic arcs for chords meeting at the ends, and braced with vertical and inclined braces. It is not capable of supporting any greater weight than a Bow String Truss of equal depth and length, and practically possesses many disadvantages.

215. Longitudinal Chord Strains.—In Fig. (82)

Let $l =$ the length of the truss,

$d =$ the depth of the truss at the centre,

$p =$ the horizontal length of a panel,

$p' =$ the length of any arc member,

$x =$ the horizontal distance of any vertical brace from the abutment,

$y =$ the depth of the truss at any point x,

$w =$ the weight upon the truss uniformly distributed.

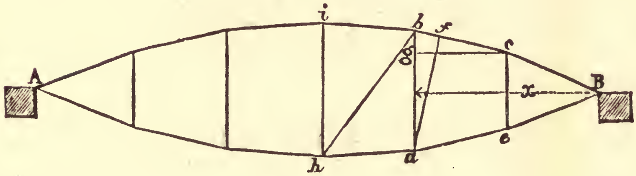


Fig. 82.

Taking moments around a in the lower chord distant horizontal x from the abutment B, we have,

$$L\,(af) = \frac{wx}{2} - \frac{wx^2}{2l}. \quad - \quad - \quad - \qquad (337)$$

The versed sine of either arc is $\frac{d}{2}$, therefore Eq. (283) the vertical height of any panel point in the upper chord above the abutments is,

$$\frac{2dx(l-x)}{l^2}, \quad - \quad - \quad - \quad - \quad - \qquad (338)$$

and the vertical depth of any panel point in the lower chord below the abutment in the same vertical section is also $\frac{2dx(l-x)}{l^2}$, whence the vertical depth of the truss at any panel point distant x from the abutment is, as in the case of the Parabolic Bow String Truss, $\frac{4dx\,(l-x)}{l^2}$.

From similar triangles,

$$af : ab :: gc : bc,$$

or substituting the values of these quantities,

$$af : \frac{4dx(l-x)}{l^2} : p : p',$$

whence,

$$af = \frac{4dx(l-x)p}{p'l^2}.$$

Substituting this value in Eq. (337), we obtain

$$L = \frac{wlp'}{8dp}. \qquad (339)$$

Equally true for either chord, and containing but one variable p'.

216. Vertical Strain in the Vertical Braces.—The depth of the truss at any panel end, ab, distant x from the abutment, is $\frac{4dx(l-x)}{l^2}$; at the next panel end towards this abutment, ce, where the distance is $x-p$, it is $\frac{4d(x-p)(l-x+p)}{l^2}$.

Subtracting these quantities, we have

$$\frac{4dp\,(l-2x+p)}{l^2}, \qquad (340)$$

for the difference in the depths of the truss at the two ends of a panel, or in the lengths of ab and ce. Half of this quantity is consequently the vertical extent of each chord member of this panel, or b is vertically one-half of this distance above c, and a vertically one-half the same beneath e. Since $bc : bg$, so is the longitudinal strain in bc to its vertical component, we have,

$$p' : \frac{2dp(l-2x+p)}{l^2} :: \frac{wlp'}{8dp} : \mathrm{V} = \frac{w}{4} - \frac{wx}{2l} + \frac{wp}{4l}. \quad (341)$$

Its horizontal component is a constant $\frac{wl}{8d}$, the lower chord member in the same panel has the same vertical component. Hence each arc or chord member supports one-half the weight that comes upon the truss between its end towards the centre and the centre, and one-fourth of a panel load; and the horizontal component remaining uniform, there is no strain upon the inclined braces. If x, in Eq. (341), be made $= x - p$, we shall obtain the vertical component of the strain in the next chord member towards the abutment, and if this be subtracted from Eq. (341), with x unchanged, the remainder is $\frac{wp}{2l}$, a constant. Therefore, at each panel point, as we pass from the centre towards the abutment, there is an increase in the weight borne by each chord of half a panel load; or each chord bears half the load; and if the whole load come first upon one chord, only one-half of it is taken by that chord, while the other half is transmitted by the vertical braces to the other chord; hence, when the truss is fully loaded, each brace is subject to a strain of $\frac{wp}{2l}$.

217. Vertical Strains in the Braces from a Moving Load.—It is evident that as the two chords have, at the two ends of any panel, the same difference in their vertical distances, as there is in the case of the Parabolic Bow String, they can take no more of the vertical strain from the moving load. Taking moments around any panel point in either chord distant x from the abutment, x and u being measured from the same abutment and x being less than u; $l-u$ being the length of a partial load, and w' being the weight of a full load of equal density with the partial load, the strain in either chord is,

$$\mathrm{L}\frac{4dx(l-x)p}{l^2p'}=\frac{w'(l-u)^2x}{2l^2},$$

whence

$$\mathrm{L}=\frac{w'(l-u)^2p'}{8d(l-x)p}. \quad - \quad - \quad - \quad - \qquad (342)$$

For simplicity take the horizontal component of this strain, which is

$$\mathrm{H}=\frac{w'(l-u)^2}{8d(l-x)}. \quad - \quad - \quad - \quad - \qquad (343)$$

A strain increasing with x or greatest at the end of the load. At x'

$$\mathrm{H}=\frac{w'(l-u)^2}{8d(l-x')}, \quad - \quad - \quad - \quad - \qquad (344)$$

Subtracting,

$$\mathrm{H}-\mathrm{H}'=\frac{w'(l-u)^2(x-x')}{8d(l-x)(l-x')},$$

is the horizontal component of the strain in the inclined brace whose ends are distant x and x' from the abutment towards which the brace leans ( ), or in ac. The vertical extent of this brace is one-half of ce, the depth of the truss at x' and one-half of ba the depth at x, its horizontal extent is p; hence

$$p : \frac{2dx(l-x)}{l^2} + \frac{2dx'(l-x)}{l^2} :: \frac{w'(l-u)^2(x-x)}{8d(l-x)(l-x')} : \mathrm{V}$$

$$= \frac{w'(l-u)^2}{4l^2}\left\{\frac{2x}{l-x} - \frac{p(l+p)}{(l-x)(l-x+p)}\right\}, \quad (345)$$

$x - p$ being substituted for x', for the vertical component of the strain in any inclined brace from the moving load, when the lower end of the brace is distant x horizontally from the abutment towards which it leans. This strain is evidently greatest when the load extends from the opposite abutment to that from which x is measured, and covers the panel point, distant x from the abutment. This being a simple truss, $\frac{w'(l-u)^2}{2l^2}$, the reaction of the unloaded abutment must be changed for the reasons explained in (54) to $\frac{w'n^2p^2}{2l(l-p)}$; and since n, the number of loaded panels in $l-u$, equals $\frac{l-x}{p}$, this quantity becomes $\frac{w'(l-x)^2}{2l(l-p)}$. Substituting this in Eq. (345) we obtain

$$\mathrm{V} = \frac{w'(l-x)}{4l(l-p)}\left\{2x - \frac{p(l-p)}{(l+p-x)}\right\}, \quad (346)$$

for the maximum vertical component of the strain in any inclined brace whose lower end is distant x from the abutment towards which it leans.

This strain is evidently always tension, and it produces compression in the vertical brace to whose upper end it is attached. This compression is greatest when the tension in the inclined brace is greatest, or when the load extends from one abutment and covers the next panel point to this brace.

Referring to Fig. (82), let the load extend from the abutment A so that the point h is loaded and a outside the load. It is evident that the vertical strains in the chord member ib and the inclined brace hb, compression in the former and tension in the latter, produce all the vertical strain that may exist in the chord member bc and the vertical brace ba; or the vertical strain in ib and hb equals the vertical strain in ba and bc; hence, if we subtract the vertical strain in bc from the sum of the strains in ib and bh, we have for the remainder the vertical strain in ba. Let V, Eq. (345), be the vertical strain in ib, x being the horizontal distance from the abutment B to point h. From moments around h,

$$\mathrm{L} = \frac{w'(l-u)^2 p'}{8d(l-x)p}, \qquad (347)$$

the reaction of the abutment being $\dfrac{w'(l-u)^2}{2l^2}$; the vertical component of this from the proportion of its length to its vertical extent,

$$p' : \frac{2dp(l-2x+p)}{l^2} :: \frac{w'(l-u)^2 p'}{8d(l-x)p} : \mathrm{V}$$

$$= \frac{w'(l-u)^2(l-2x+p)}{4l^2(l-x)}. \qquad (348)$$

Adding Eqs. (345) and (348), we have,

$$V = \frac{w'(l-u)^2}{4l^2}\left\{\frac{2x}{l-x} - \frac{p(l+p)}{(l-x)(l-x+p)} + \frac{l-2x+p}{l-x}\right\} \quad (349)$$

which readily reduces to

$$V = \frac{w'(l-u)^2(l+p)}{4l^2(l-x+p)}, \quad - \quad - \quad - \quad (350)$$

for the total vertical strain coming upon the point b, and taken by the members bc and ba. If x of Eq. (348) be changed to $x - p$, we shall have for the vertical strain in bc,

$$V = \frac{w'(l-u)^2(l-2x+3p)}{4l^2(l-x+p)}. \quad - \quad - \quad (351)$$

Subtract Eq. (351) from Eq. (350) and

$$V = \frac{w'(l-u)^2 2(x-p)}{4l^2(l-x+p)} \quad - \quad - \quad - \quad (352)$$

is the remainder and the strain in ba. In this equation the reaction of the abutment $\frac{w'(l-u)^2}{2l^2}$ must be changed to $\frac{w'n^2p^2}{2l(l-p)}$, and since x is the distance to the panel point beyond the vertical brace to which the equation refers, let it be changed to $x' + p$, so that x' shall be the distance to the brace. Then, since $n = \frac{l-x'-p}{p}$, making these changes, Eq. (352) becomes

$$V = \frac{w'(l-x'-p)^2x'}{2l(l-p)(l-x')}. \quad - \quad - \quad - \quad (353)$$

The vertical braces are subject to the action of the load moving in either direction, or a strain may come upon them from either of the ties attached to their upper ends. The greatest strain is when the load covers more than half the truss, or when x' does not exceed $\frac{l}{2}$. If the roadway be upon the upper chord the proportion of the compression from the constant load must be added to Eq. (353). If upon the lower chord the tension from the constant load must be deducted.

218. Example.—In Fig. (81),

Let $l = 350$ feet, the length of the truss,

$d = 49$ feet, the depth of the truss at the centre,

$p = 25$ feet, the horizontal length of a panel,

$w' = 350$ tons, the weight of the full movable load,

$w = 175$ tons, the constant truss weight.

Substituting values in Eq. (339) we have the following table of chord strains, w of this equation being the maximum load, 525 tons:

Values of $p.'$	25.83	25.60	25.40	25.24	25.12	25.04	25.01
Strains in Tons.	484.31	480.00	476.25	473.25	471.00	469.50	468.94
Compression in	AB & OP	BC & NO	CD & MN	DE & LM	EF & KL	FG & IK	GH & HI
Tension in	Ab & oP	bc & no	cd & mn	de & lm	ef & kl	fg & ik	gh & hi

Substituting values in Eq. (346), and multiplying the vertical strain by the secants of the inclined brace angles, we may form the following table:

Values of x.	Strains in Tons.	Tension in
50	30.96	Bc & nO
75	33.41	Cd & mN
100	38.68	De & lM
125	40.32	Ef & kL
150	45.13	Fg & iK
175	45.94	Gh & hI
200	44.99	Hi & gH
225	42.31	Ik & fG
250	37.93	Kl & eF
275	31.90	Lm & dE
300	24.30	Mn & cD
325	14.95	No & bC

If three-fourths of the constant load be considered as concentrated upon the lower chord, which also receives the moving load (the upper chord bearing directly the other fourth), each vertical brace will be subject to a tension of one-fourth of a panel load from the constant load, or $\frac{wp}{4l}$, and one-half of a panel load of the moving load, or $\frac{w'p}{2l}$; hence, $\frac{wp}{4l}+\frac{w'p}{2l}=15.625$ tons is the maximum tension in the vertical braces.

Substituting values in Eq. (353), and deducting from the results the constant load tension of $\frac{wp}{4l}=3.125$, the following table of maximum compressions in the vertical braces may be formed:

Values of x.	25	50	75	100	125	150	175
Strains in Tons.	7.53	16.27	23.10	28.03	31.07	32.21	31.49
Compression in	Bb & Oo	Cc & Nn	Dd & Mm	Ee & Ll	Ff & Kk	Gg & Ii	Hh

CHAPTER XI.

THE KUILENBERG TRUSS.

219.—The plan of this truss is shown in Fig. (83).

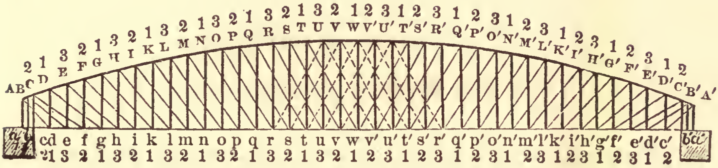


Fig. 83.

For trusses of this character, having the upper chord arched, but not meeting the lower and horizontal chord at the abutments, equations may be prepared, containing but one variable, as in the previous cases; but in this case to express the depth of the truss at any point in terms of the distance of that point from the abutment, would give a long and somewhat intricate equation. For this reason, and to give an example of the manner in which any form of truss may be analyzed, and its strains determined, only the simplest forms of equations will be used.

220.—Let $l = 152$ metres, the distance between the abutments,

$d = 20$ metres, the depth of the truss at the centre,

$p = 4$ metres, the length of a panel,

$n = 1.2$ metres, the distance on the abutment between the end posts,

*$w' = 500$ tons, the full movable load,

$w = 500$ tons, the constant truss weight, uniformly distributed,

$x =$ the horizontal distance of a panel point from the abutment not from the end of the truss,

$y =$ the vertical depth of the truss at x.

The truss extends beyond the abutment at either end a certain distance $= 2n$, $ab = bc = n$. The upper chord is the arc of a circle whose diameter $= 493.33$ metres, and the length of the end post $= 8$ metres.

221. Horizontal Strains.—Under the full load, in which case the chord strains are greatest, this truss, like the trusses with horizontal chords, may be divided into three simple trusses,

No. 1.—Fig. (84).

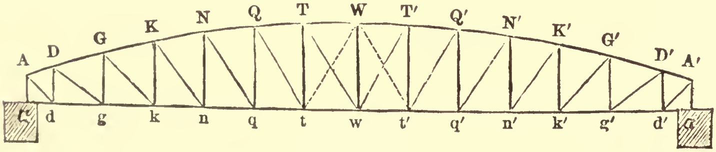


Fig. 84.

* These weights are assumed.

No. 2—Fig. (85).

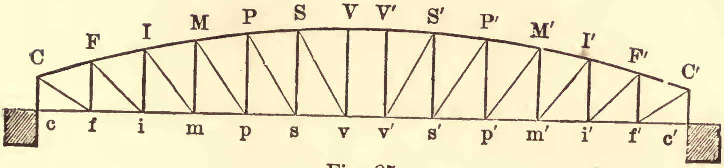


Fig. 85.

No. 3.—Fig. (86).

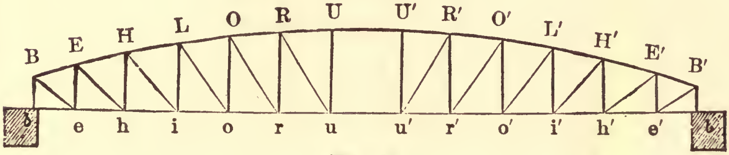


Fig. 86.

Simple Truss No. 1 bears $\frac{1}{3}\left(w + \frac{wp}{l}\right)$, ($w$, for convenience, expressing the full load and the truss weight), therefore the moment of the reaction of the abutment upon it is at any point x, since its end post is $2n$ beyond the abutment,

$$\left(\frac{w}{6} + \frac{wp}{6l}\right)(x + 2n),$$

and the moment of the load on x is,

$$\frac{1}{3}\left(\frac{wx}{2l} + \frac{wpx}{2l} - \frac{wp^2}{l}\right).$$

Hence H, the horizontal component of the compression in the upper chord, or the tension in the lower chord is,

$$Hy = \frac{wx}{6} - \frac{wx^2}{6l} + \frac{wp^2}{3l} + \frac{wn}{3} + \frac{wpn'}{3l};$$

whence,

$$H = \frac{w}{6ly}(lx - x^2 + 2p^2 + 2ln + 2pn). \qquad (354)$$

Substituting values given above, w being 1000 tons,

$$H = \frac{125x[(152-x)+406.4]}{114.y};$$

Whence the following table of chord strains in Simple Truss No. 1:

Values of x.	Values of y.	Horizontal Strains in Tons.	Horizontal Component of Compression in	Tension in
4	9.92	110.34	AD & A′D′	dg & d′g′
16	13.04	217.14	DG & D′G′	gk & g′k′
28	15.57	273.13	GK & G′K′	kn & k′n′
40	17.52	305.81	KN & K′N′	nq & n′q′
52	18.9	325.25	NQ & N′Q′	qt & q′t′
64	19.72	335.75	QT & Q′T′	tw & t′w′
76	20	338.95	TW & T′W′	

There being no tension in *ad*.

Simple Truss No. 2 bears $\frac{1}{3}\left(w+\frac{wp}{l}\right)$, and ending at the abutment, the moment of the abutment reaction is,

$$\frac{wx}{6}-\frac{wpx}{6l},$$

the load on x is $\frac{wx^2}{6l}$, hence

$$H = \frac{w}{6ly}(lx - x^2 + px). \quad - \quad - \quad (355)$$

Substituting values, w being put for $(w' + w_,)$

$$H = \frac{250}{203y}\left\{x(156 - x)\right\}$$

whence the following table of chord strains in Simple Truss No. 2:

Values of x.	Values of y.	Strains in Tons.	Horizontal Component of Compression in	Tension in
12	12.05	157.24	CF & C′F′	fi & f′i′
24	14.79	234.86	FI & F′I′	im & i′m′
36	16.93	279.79	IM & I′M′	mp & m′p′
48	18.5	307.25	MP & M′P′	ps & p′s′
60	19.51	323.72	PS & P′S′	sv & s′v′
72	19.97	332.07	SW & S′W′	

No strain in *cf.*

Simple Truss No. 3 bears $\frac{1}{3}\left(w + \frac{wp}{l}\right) - \frac{wp}{l}$, and since its end post is distant n beyond the abutment, the moment of the abutment reaction is $\left(\frac{w}{6} - \frac{wp}{6l}\right)(x+n)$, the moment of the load on x is $\frac{wx^2}{6l} - \frac{wpx}{6l} - \frac{wp^2}{3l}$; hence,

$$Hy = \left(\frac{w}{6} - \frac{wp}{3l}\right)(x + n) - \frac{wx^2}{6l} + \frac{wpx}{6l} + \frac{wp^2}{3l};$$

$$\therefore \quad H = \frac{w}{6ly}\left\{ lx - x^2 - px + 2p^2 + nl - 2pn \right) ; \quad (356)$$

substituting values,

$$H = \frac{250}{203y}\left\{ x(148 - x) + 204.8 \right\},$$

whence the following table of chord strains in Simple Truss No. 3:

Values of x.	Values of y.	Strains in Tons.	Horizontal Component of Compression in	Tension in
8	11.03	131.7	BE & B'E'	eh & e'h'
20	13.95	217.25	EH & E'H'	hl & h'l'
32	16.28	263.80	HL & H'L'	lo & l'o'
44	18.04	290.57	LO & L'O'	or & o'r'
56	19.24	305.29	OR & O'R'	ru & r'u'
68	19.88	311.34	RW & R'W'	

No strain in *be*.

These are the horizontal strains in the chords of the Simple Trusses. The strain in the Compound Truss in any member is the sum of the strains in the Simple Trusses affecting that member; for example, in the upper chord the strain in HI is the sum of the strains in HL in No. 3, in GK in No. 1, and in FI in No. 2. In the lower chord the strain in *hi* is the sum of the strains in *hl*, *gk*, and *fi*. Consequently we may arrange a table as follows: put the panel points, beginning at the end of the truss, in a vertical column; opposite to and in line with each point, place the strain extending from that point to the next point towards the centre of the same simple truss; then at any point the sum of the simple truss strain at that point, and the two strains immediately above it, is the total strain in that member of the upper chord whose abutment end is at the point referred to. Similarly of the lower chord tensions, thus:

Upper Chord Panel Points.	Lower Chord Panel Points.	Simple Truss Strains.	Compound Truss Strains.	Horizontal Component of Compression in	Tension in
A	d	110.34	110.34	AB & A'B'	de & d'e'
B	e	131.70	242.14	BC & B'C'	ef & e'f'
C	f	157.24	399.28	CD & C'D'	fg & f'g'
D	g	217.14	506.08	DE & D'E'	gh & g'h'
E	h	217.25	591.63	EF & E'F'	hi & h'i'
F	i	234.86	669.25	FG & F'G'	ik & i'k'
G	k	273.13	725.24	GH & G'H'	kl & k'l'
H	l	263.80	771.79	HI & H'I'	lm & l'm'
I	m	279.79	816.72	IK & I'K'	mn & m'n'
K	n	305.81	849.40	KL & K'L'	no & n'o'
L	o	290.57	876.17	LM & L'M'	op & o'p'
M	p	307.25	903.63	MN & M'N	pq & p'q'
N	q	325.25	923.07	NO & N'O'	qr & q'r'
O	r	305.29	937.79	OP & O'P'	rs & r's'
P	s	323.72	954.26	PQ & P'Q'	st & s't'
Q	t	335.75	964.76	QR & Q'R'	tu & t'u
R	u	311.34	970.81	RS & R'S'	uv & u'v'
S	v	332.07	979.16	ST & S'T'	vw & v'w
T		338.95	982.36	TU, UV, VW, T'U', U'V' & V'W	

Multiplying the horizontal components of the compressions in the upper chord by the length of the member, and dividing by the horizontal length of the panels, we obtain the following table of upper chord compressions:

AB & A'B'	BC & B'C'	CD & C'D'	DE & D'E'	EF & E'F'	FG & F'G'	GH & G'H'	HI & H'I'	IK & I'K'	KL & K'L'	LM & L'M'
119.54	262.32	413.25	523.79	610.87	691.60	747.00	793.01	837.12	868.51	893.69

(*Continued*)

MN & M'N'	NO & N'O'	OP & O'P'	PQ & P'Q'	QR & Q'R'	RS & R'S'	ST & S'T'	TU & T'U'	UV & U'V'	VW & V'W'
921.70	939.22	951.86	966.19	974.41	980.52	986.50	987.77	984.82	982.36

The excess of horizontal strain in both chords of No. 1, caused by this objectionable extension of the truss beyond the abutments, is the quantity $\frac{w}{3ly}(ln + pn)$, amounting to 20.53 tons at the centre, increasing to 41.38 tons at the abutments. In No. 3 this excess, $\frac{w}{6ly}(nl - 2pn)$, is 10. tons at the centre and 18.45 tons at the abutments, making a total unnecessary excess of strain of 30.53 tons at the centre, increasing to 59.83 tons at the abutments.

222. Vertical Strains from the Permanent Load.—The above tables show that under a full load the chord strains increase from the abutments to the centre, and consequently the vertical braces are struts and the inclined braces ties. The figures of the Simple Trusses, (84), (85), and (86), show all the braces needed under a full or constant uniform load. If the truss supports only its own weight, w, the strains in the chords will be only one-half those given in the tables, since w is one-half of $w' + w$; and the horizontal component of the strain in any brace (Kn, of Simple Truss No. 1, for example), from the constant load, w, will be the difference between the horizontal components of the strains in GK and KN, or, what is the same, between the strains in kn and nq; hence $3p : y :: \text{H} - \text{H}' : \text{V}$, or the horizontal extent of any brace is to its vertical extent as the horizontal component of the strain in that brace is to its vertical component. And the compression from the same load in any vertical brace is equal to the vertical strain in the tie which meets that brace at the lower chord, less a panel weight of that load, $\frac{w'p}{l}$, the load being considered as concentrated at the lower chord panel points.

In this manner the following tables of the strains in the vertical struts and of the vertical components of the tensions in the inclined ties from the permanent load w are formed:

SIMPLE TRUSS NO. 1.

Strains in Tons.	68.96	44.14	30.42	21.20	14.19	8.27	2.63	4.89	7.90
Tension in	Ad & A'd'	Dg & D'g'	Gk & G'k'	Kn & K'n	Nq & N'q'	Qt & Q't'	Tw & T'w	Tt & T't'	Ww
Strains in Tons.	85.51	55.8	30.98	17.26	8.04	1.03			
Compression in	Aa & A'a'	Dd & D'd'	Gg & G'g'	Kk & K'k'	Nn & N'n'	Qq & Q'q'			

As the vertical strain in Qt is 8.27, and the load upon point t is $\frac{w'p}{l} = 13.16$, the strain in Tt is tension, and its amount is $13.16 - 8.27 = 4.89$. The vertical strains in Tw and T'w $= 5.26$; the load at $w = 13.16$; hence $15.16 - 5.26 = 7.90$ tension in Ww.

SIMPLE TRUSS NO. 2.

Strains in Tons.	57.32	38.97	27.68	19.37	12.69	6.78	0.47	6.37
Tension in	Cf & C'f'	Fi & F'i'	Im & I'm'	Mp & M'p'	Ps & P's'	Sv & S'v'	Ss & S's'	Vv & V'v'
Strains in Tons.	78.94	44.16	25.81	14.52	6.21			
Compression in	Cc & C'c'	Ff & F'f'	Ii & I'i'	Mm & M'm'	Pp & P'p'			

There is tension in Ss and Vv, as explained after the previous table, because the inclined ties from their lower chord points do not support a panel load.

SIMPLE TRUSS No. 3.

Strains in Tons.	59.91	39.32	27.02	18.16	11.06	4.85	2.10	8.31
Tension in	Be & B′e′	Eh & E′h′	Hl & H′l′	Lo & L′o′	Or & O′r′	Ru & R′u′	Rr & R′r′	Uu & U′u′
Strains in Tons.	78.94	46.75	26.16	13.86	5.00			
Compression in	Bb & B′b′	Ee & E′e′	Hh & H′h′	Ll & L′l′	Oo & O′o′			

The cause of the tension in Rr and Uu was explained before; the compression in each end post of any simple truss is obviously half the weight borne by that truss.

223. Vertical Strains from a Moving Load.—Let $l-u$ represent the length of the moving load as before, extending from one abutment to the centre of a panel of Simple Truss No. 1; then the load on $l-u$ is $\frac{1}{3}\frac{w'}{l}\left(l-u+\frac{p}{2}\right)$; divide this by $l+4n$, which is the length of the leverage of this simple truss, and multiply by $\frac{1}{2}\left(l-u-\frac{p}{2}\right)+2n$, the distance of the centre of gravity of the load from the loaded abutment, and we have

$$V=\frac{w'}{6l(l+4n)}\left\{(l-u)^2-\frac{p^2}{4}+4n\left(l-u+\frac{p}{2}\right)\right\}, \quad (357)$$

for the reaction of the unloaded abutment upon this simple truss.

Taking moments around any panel point of this truss outside the load, x and u being measured from the same abutment, we have,

$$H = \frac{Vx}{y} \quad - \quad - \quad - \quad - \quad - \qquad (358)$$

for the tension in the lower chord and the horizontal component of the compression in the upper chord. And at the next panel point of the same truss towards the abutment from which x is measured,

$$H' = \frac{V(x - 3p)}{y'}. \quad - \quad - \quad - \quad - \qquad (359)$$

Subtracting Eq. (359) from Eq. (358)

$$H - H' = \frac{[V(x(y' - y) + 3py]}{yy'}, \quad - \qquad (360)$$

is the horizontal component of the brace which extends from x to $x - 3p$, and which, since the horizontal strain increases towards the end of the load, is a tie, if the upper end inclines towards the abutment from which x is measured. The vertical extent of this tie is y', its horizontal extent is $3p$; hence, multiplying Eq. (360) by y', the vertical extent of this tie, and dividing by its horizontal extent, $3p$, we obtain

$$V\left(\frac{x(y' - y)}{3py} + 1\right) \quad - \quad - \quad - \qquad (361)$$

for the vertical component of the strain in any inclined brace, outside the load, from a moving load, whose lower chord end is distant x from the unloaded abutment, whose vertical extent is y', y being the vertical height of the truss at x. Substituting for V its value given by Eq (357), we have

$$\frac{w'}{6l(l+4n)}\left\{(l-u)^2-\frac{p^2}{4}+4n\left(l-u+\frac{p}{2}\right)\right\}\left\{\frac{x(y'-y)}{3py}+1\right\} \quad (362)$$

Substituting the values given above, this becomes

$$\frac{(152-u)^2-4.8\,u+735.2)}{286.0032}\left\{\frac{x(y'-y)}{12y}+1\right\} \qquad (363)$$

The greatest strain from this equation upon any inclined tie is when the load covers the point to which x is measured, or when $u=x-\frac{3p}{2}$, and this strain is greater than the strain from the full load.

To this strain from the moving load is to be added the strain from the constant load so long as the two are acting upon the same abutment; but when x of Eq. (363) becomes greater than $\frac{l}{2}$, then the strains in the same panel are acting in opposite directions, and one neutralizes its amount in the other. While Eq. (363), with x greater than $\frac{l}{2}$, is greater than the constant load vertical strain, in the same panel in which the brace from x to $x-3p$ is, a tie whose upper end inclines towards the abutment from which x is measured is needed to support the difference in these vertical strains.

In the following table the constant load vertical strains taken from the table for Simple Truss No. 1 are placed with the plus sign, as we proceed from either end towards the centre and beyond with the minus sign.

This is upon the same reasoning given before. The sum is the strain in the inclined brace.

Values of u.	Values of x.	Values of y.	Values of y'.	Strains in Tons from Moving Load.	Constant Load Vertical Strains.	Total Strains in Tons.	Tension in
10	16	13.04	9.92	+49.65	+44.14	+93.79	Dg & D'g'
22	28	15.57	13.04	+38.06	+30.42	+68.48	Gk & G'k'
34	40	17.52	15.57	+31.88	+21.20	+53.08	Kn & K'n'
46	52	18.9	17.52	+28.11	+14.19	+42.30	Nq & N'q'
58	64	19.72	18.9	+25.28	+ 8.27	+33.55	Qt & Q't'
70	76	20.	19.72	+22.72	+ 2.63	+25.35	Tw & T'w
82	88	19.72	20	+20.16	− 2.63	+17.53	Wt' & Wt
94	100	18.9	19.72	+17.83	− 8.27	+ 9.56	

There is no inclined brace from T' to q' or from T to q, consequently the strain, 9.56 tons, is compression in the braces with the opposite inclinations, Q't' or Qt, with the moving load assumed, which is certainly not too great. When $u = 106$, the strain from the moving load to the abutment from which it is measured, is less than the permanent strain in the same panel to the opposite abutment, and consequently no further counter-bracing is necessary.

Subtracting from these strains the weight of a panel load permanent and movable, $\frac{(w'+w)p}{l} = 26.32$ tons, we obtain the compression in the vertical braces connected with the lower ends of the ties, as follows:

Strains in Tons.	67.46	42.16	26.76	15.98	7.23
Compression in	Gg & G′g′	Kk & K′k′	Nn & N′n′	Qq & Q′q′	Tt & T′t′

No compression in Ww.

Under the moving load one-half of Simple Truss No. 2 is united by the counterbraces with the opposite half of Simple Truss No. 3. Let the load be considered as extending from the abutment end of Simple Truss No. 3, or from left end of Fig. (87).

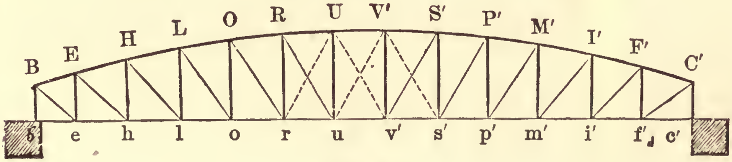


Fig. 87.

The load on $l-u$ is, u being distances from the right abutment to the centres of the panels of Simple Truss No. 2 while less than $\frac{l}{2}$, and when greater than $\frac{l}{2}$, to the centres of the panels of Simple Truss No. 3,

$$\frac{1}{3}\frac{w'}{l}\left(l-u-\frac{p}{2}\right).$$

Dividing by the length of the leverage of the halves of these two trusses, $l + n$, and multiplying by $\frac{1}{2}\left(l - u + \frac{p}{2}\right) + n$, the distance of the centre of gravity of the load from the loaded abutment, we have

$$\frac{w'}{6l(l+n)}\left\{(l-u)^2 - \frac{p^2}{4} + 2n\left(l - u + \frac{p}{2}\right)\right\}, \quad (364)$$

for the reaction of the unloaded abutment upon Simple Truss No. 2 when u is less than $\frac{l}{2}$, and upon No. 3 when u is greater than $\frac{l}{2}$.

Substituting values in Eq. (364), we obtain by the same proccss of reasoning as in the case of Simple Truss No. 1,

$$V = \frac{(156 - u)^2 - 2.4\,u + 356}{279.4368}\left\{\frac{x(y' - y)}{12y} + 1\right\}, \quad (365)$$

for the vertical component of the strain, from the moving load, in any brace of Simple Truss No. 2 whose lower chord end is distant x from the abutment towards which the brace leans, while x does not exceed $\frac{l}{2}$, and beyond that point in the counterbraces of Simple Truss No. 3 having similar inclinations.

In the following table the strains from the moving load are from Eq. (365); the strains from the constant load are from the table given before for Simple Truss No. 2; to the centre and beyond that, where they become minus, from Simple Truss No. 3.

Values of u.	Values of x.	Values of y.	Values of y'.	Strain in Tons from Moving Load.	Constant Load Vertical Strains.	Total Strain in Tons.	Tension in
18	24	14.79	12.07	+41.33	+38.97	+80.30	Fi & F'i'
30	36	16.93	14.79	+33.61	+27.68	+61.29	Im & I'm'
42	48	18.5	16.93	+29.21	+19.37	+48.58	Mp & M'p'
54	60	19.51	18.50	+26.08	+12.69	+38.77	Ps & P's'
66	72	19.97	19.51	+23.42	+ 6.78	+30.20	Sv & S'v'
78	84	19.88	19.97	+20.84	+ 0.00	+20.84	Vu' & V'u
90	96	19.24	19.88	+18.04	− 4.85	+13.19	U'r' & Ur
102	118	18.04	19.24	+14.93	−11.06	+ 3.87	

The compression in Or and O'r', since there are no ties, Ro and R'o' is 3.87 tons. Beyond $u = 102$ no counterbracing is necessary.

Subtracting from the strains a panel load, $\dfrac{(w' + w)p}{l}$ $= 26.32$ tons, we may form the following table of compressions in the vertical braces:

Strains in Tons.	53.98	34.97	22.26	12.45	3.88	7.68
Compression in	Ii & I'i'	Mm & M'm'	Pp & P'p'	Ss & S's'	Vv & V'v'	U'u' & Uu

If the load be considered as extending from the abutment end of Simple Truss No. 2, or from right end of Fig. (87), by similar reasoning we obtain

$$V = \frac{w'}{6l(l+n)}\left\{(l-u)^2 - \frac{9p^2}{4}\right\} \quad - \quad - \quad - \quad (366)$$

for the reaction of the unloaded abutment, and substituting values,

$$\frac{(152-u)^2 - 36}{279.4368}\left\{\frac{x(y'-y)}{12y} + 1\right\} \quad - \quad - \quad (367)$$

for the vertical component of the strain from the moving load in any inclined brace of Simple Truss No. 3 whose lower chord end is distant x from the abutment towards which the brace leans, while x does not exceed $\frac{l}{2}$, and beyond that point in the counterbraces of Simple Truss No. 2, having similar inclinations.

The following table is prepared as were the previous two of the inclined brace strains:

Values of u.	Values of x.	Values of y.	Values of y'.	Strains in Tons from Moving Load.	Constant Load Vertical Strains.	Total Strain in Tons.	Tension in
14	20	13.95	11.03	+44.21	+39.32	+83.53	Eh & E'h'
26	32	16.28	13.95	+35.09	+27.02	+62.11	Hl & H'l'
38	44	18.04	16.28	+29.78	+18.16	+47.94	Lo & L'o'
50	56	19.24	18.04	+26.30	+11.06	+37.36	Or & O'r'
62	68	19.88	19.24	+23.61	+14.85	+28.46	Ru & R'u'
74	80	19.97	19.88	+20.97	O	+20.97	Uv' & U'v
86	92	19.51	19.97	+18.26	− 6.78	+12.48	V's' & Vs
98	104	18.5	19.51	+15.16	12.69	+ 2.47	

The compression in Ps and P′s′, since there are no ties, Sp and S′p′ is 2.47 tons. Beyond $w = 98$, counter-bracing is unnecessary.

Whence the compression in the vertical braces is as follows:

Strains in Tons.	57.21	35.79	21.62	11.04
Tension in	Hh & H′h′	Ll & L′l′	Oo & O′o′	Rr & R′r′

The maximum vertical compressions upon the end struts and ties of each truss, are when the truss is fully loaded, and are consequently double the strains given in the tables for the permanent load strains; the second struts receive the maximum strains at the same time. The maximum tensions in the central vertical braces also occur under the maximum load; hence the following table:

Strains in Tons.	171.02	157.88	157.88	111.60	93.50	88.32			
Compression in	Aa & A′a′	Bb & B′b′	Cc & C′c′	Dd & D′d′	Ee & E′e′	Ff & F′f′			
Strains in Tons.	137.92	119.82	114.64	4.20	0.94	9.78	16.62	12.74	15.80
Tension in	Ad & A′d′	Be & B′e′	Cf & C′f′	Rr & R′r′	Ss & S′s′	Tt & T′t′	Uu & U′u′	Vv & V′v′	Ww

The strains which have been determined for the inclined braces are the vertical components of the strains to which they are subject; hence it is necessary to divide them by the vertical extents and multiply them by the lengths of the braces, which is the same as multiplying them by the secants of the angles the braces make with a vertical line, to obtain the longitudinal strains. Performing this operation we may form the following table:

Strains in Tons.	Tension in	Strains in Tons.	Tension in
176.71	Ad & A'd'	51.28	Nq & N'q'
178.08	Be & B'e'	44.88	Or & O'r'
198.59	Cf & C'f'	46.21	Ps & P's'
144.09	Dg & D'g'	39.74	Qt & Q't'
126.39	Eh & E'h'	33.55	Ru & R'u'
113.23	Fi & F'i'	35.45	Sv & S'v'
93.06	Gk & G'k'	29.18	Tw & T'w
81.70	Hl & H'l'	24.49	Uv' & U'v
85.03	Im & I'm'	24.31	Vu' & V'u
67.02	Kn & K'n'	20.49	Wt' & Wt
59.54	Lo & L'o'	13.94	V's' & Vs
59.54	Mp & M'p'	15:41	U'r' & Ur

Strain in Tons.	4.65	2.94	11.32
Compression in	Or & O'r'	Ps & P's'	Qt & Q't'

CHAPTER XII.

THE BOLLMAN AND FINK TRUSSES.

224.—These trusses take their names from the designers, and, strictly speaking, do not come within the definition of a truss, as given in the opening chapter. Each may be considered, however, as composed of a number of simple triangular trusses, and the determination of the strains affecting them presents no difficulty.

CASE I.—THE BOLLMAN TRUSS.

225. The Bollman Truss.—Let Fig. (88) represent a truss in which the only chord, AB, is horizontal, and in

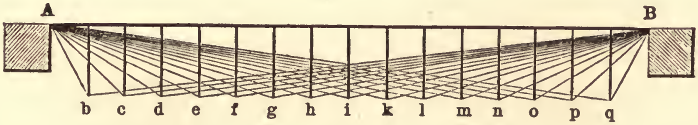


Fig. 88.

which the inclined braces meet the chord at its ends only. It is evident that the strain affecting this chord is compression, and that it is uniform throughout the length of the chord, for no brace whose strain has a horizontal component meets the chord except at its ends, and that it is greatest when the truss is fully loaded.

226. Horizontal Strains.

Let l = the length of the truss,
d = the depth of the truss, or the length of the vertical braces,
p = the length of a panel, or the distance between the vertical braces,
w = the maximum uniform weight, both fixed and movable.

Taking moments around a point in the centre of truss, and in the line of the lower ends of the vertical braces, the strains in the inclined braces crossing the centre may be disregarded, as their amounts in opposite directions exactly balance each other, and we have, therefore,

$$H = \frac{wl}{8d}. \quad - \quad - \quad - \quad - \quad - \quad (366)$$

is the strain in the horizontal chord.

227. Vertical Strains in the Vertical Braces.—The vertical braces are struts, and each one can only support the maximum panel load, for no brace connects with its upper end to bring any greater vertical strain upon it; hence its greatest strain is when it is fully loaded either from a full truss load or from a moving load; therefore

$$V = \frac{wp}{l} \quad - \quad - \quad - \quad - \quad - \quad (367)$$

is the maximum strain in each vertical brace.

228. Vertical Strains in the Inclined Braces.—Each vertical brace, with the inclined braces attached to its lower end, is, with the horizontal chord, a simple triangular truss.

Let x be the distance of a vertical strut from one abutment; then by the principles of the lever, since $\frac{wp}{l}$ is the load upon the strut, $\frac{wp}{l} \div l \times (l - x)$

$$= \frac{wp}{l} - \frac{wpx}{l^2}, \qquad - \quad - \quad - \quad - \qquad (368)$$

is the reaction of that abutment from which x is measured upon the inclined brace from the lower end of the strut to which x is measured, and consequently the vertical component of the strain, which is tension, in that inclined brace. Similarly,

$$V = \frac{wpx}{l^2}, \qquad - \quad - \quad - \quad - \quad - \qquad (369)$$

is the vertical component of the strain in the other inclined brace from the same strut. One equation only, however, is needed.

Dividing these equations by d, the depth of the truss, or the vertical extent of the inclined tie, and multiplying by the length of the tie itself, we obtain the longitudinal strain.

229. Example.—In Fig. (88),

Let $l = 160$ feet, the length of the truss or chord,
$d = 15$ feet, the depth of the truss,
$p = 10$ feet, the length of a panel,
$w = 240$ tons, the full weight of the truss and of the load.

Therefore $\frac{wl}{8d} = \frac{240 \times 160}{8 \times 15} = 320$ tons, compression throughout the horizontal chord. And $\frac{wp}{l} = \frac{240 \times 10}{160}$ $= 15$ tons, compression in each vertical strut; and $\frac{wp}{l} - \frac{wpx}{l^2} = 15 - \frac{3x}{32}$.

Substituting values of x, dividing by d, and multiplying by the lengths of the ties, we have the following table of tensions in the ties:

Values of x.	Strains in Tons.	Tension in
10	16.09	bA & qB
20	21.36	cA & pB
30	27.25	dA & oB
40	32.04	eA & nB
50	35.89	fA & mB
60	38.65	gA & lB
70	40.26	hA & kB
80	40.71	iA & iB
90	39.92	kA & hB
100	37.92	lA & gB
110	34.69	mA & fB
120	28.23	nA & eB
130	24.54	oA & dB
140	17.6	pA & cB
150	9.42	qA & bB

230. A Truss Containing an odd number of Panels.—This truss may be divided into an odd number of panels or chord members, in which case, by taking moments as before, we obtain for the chord strain,

$$H = \frac{wl}{8d} - \frac{wp^2}{8dl}. \quad - \quad - \quad - \quad - \qquad (370)$$

The strains in the braces are found from the same equations as in the previous case.

231.—In practice, in the Bollman Truss, there are two light rods from the bottom of each truss to the tops of the next on either side, and a horizontal rod connecting the lower ends of the struts.

CASE II.—THE FINK TRUSS.

232. The Fink Truss.—In this truss, shown in Fig. (89), as in the Bollman, the single chord is subject to compression, the vertical braces are struts, and the inclined braces ties.

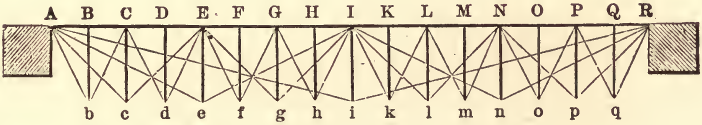


Fig. 89.

233. Vertical Strains.—The struts attached to the chord at the points B, D, F, H, K, M, O, and Q, or the alternate struts beginning with those next the abutments, each bear one panel load, since, as no brace meets them or is connected with them at the chord, they can receive no weight or strain except from the load immediately upon them. The braces from their lower ends having equal inclinations receive equal amounts of vertical strain; or

Let $l =$ the length of the truss,
$p =$ the length of a panel,
$w =$ the uniform maximum load,

then $\frac{wp}{l}$ is the weight upon each of these struts, and $\frac{wp}{2l}$ the vertical component of the tension in each of the ties from their lower ends, or in all the ties in the truss having a horizontal extent of p, or a panel length.

The struts from the points C, G, L, and P, have, in in addition to the panel load upon them, a half panel load from each of the inclined braces which meet them at the chord, and consequently are subject to a vertical strain of $\frac{2wp}{l}$; the inclined braces from their lower ends therefore have each a vertical strain $\frac{wp}{l}$, and a horizontal extent of two panel lengths, or $2p$; whence we form this rule.

The vertical component of the strain in any tie equals half the panel loads in its horizontal extent; thus nR has a horizontal extent of four panels, the vertical component of its strain is therefore $\frac{2wp}{l}$; iR has a horizontal extent of eight panels, the vertical component of its strain is therefore $\frac{4wp}{l}$; or let n = number of panels in the horizontal extent of a tie, then $\frac{wpn}{2l}$ is the vertical component of the strain in that tie, and this divided by the depth of the truss, and multiplied by the length of the tie, will give the longitudinal strain.

Evidently $\frac{wpn}{l}$ is the compression in the strut to whose lower end the tie, having a horizontal extent of n, is attached.

234. Horizontal Strains.—The strain in the chord from any tie having a horizontal extent of p, or the horizontal component of the strain in that tie, is

$$d : p :: \frac{wp}{2l} : \frac{wp^2}{2dl}.$$

From the ties whose horizontal extent = $2p$,

$$d : 2p :: \frac{wp}{l} : \frac{2wp^2}{dl}.$$

From the ties whose horizontal extent = $4p$,

$$d : 4p :: \frac{2wp}{l} : \frac{8wp^2}{dl};$$

and from those whose horizontal extent = $8p$,

$$d : 8p :: \frac{4wp}{l} : \frac{32wp^2}{dl}.$$

It is evident that all these strains affect each member of the chord, and that, therefore,

$$\frac{wp^2}{2dl} + \frac{2wp^2}{dl} + \frac{8wp^2}{dl} + \frac{32wp^2}{dl} = \frac{42\frac{1}{2}wp^2}{dl}$$

is the uniform compression throughout the chord of a Fink Truss, containing sixteen panels.

It will be noticed that general equations are not given for this truss, but special equations are determined for a truss divided, as shown in the figure; and for a truss containing a different number of panels a different horizontal equation must be prepared. It will also be noticed that the greatest strains are from the full load.

235. Example.—In Fig. (89),

Let $l = 160$ feet, the length of the truss,

$d = 15$ feet, the depth of the truss,

$p = 10$ feet, the length of a panel,

$w = 240$ tons, the maximum uniform load.

Hence

$$\frac{42\frac{1}{2}wp^2}{dl} = \frac{42\frac{1}{2} \times 240 \times 10 \times 10}{15 \times 160} = 425 \text{ tons},$$

compression throughout the upper chord.

For the struts, $\dfrac{wpn}{l} = 15\,n$, whence,

Values of n.	1	2	4	8
Strains in Tons.	15	30	60	120
Compression in	Bb, Dd, Ef, Hh, Kk, Mm, Oo, & Qq	Cc, Gg, Ll, & Pp	Ee & Nn	Ii

For the tension in the ties, $\frac{wpn}{2l}$, divided by d and multiplied by their lengths, we have,

Values of n.	1	2	4	8
Strains in Tons.	9.01	24.41	85.44	325.66
Tension in	Ab, Cb, Cd, Ed, Ef, Gf, Gh, Ih, Ik, Lk, Lm, Nm, No, Po, Pq, & Rq.	Ac, Ec, Eg, Ig, Il, Nl, Np, & Rp	Ae, Ie, In, & Rn	Ai & Ri

CHAPTER XIII.

RESISTANCE OF MATERIALS TO COMPRESSION AND TENSION.

I.—COMPRESSION.

236.—Mr. Eaton Hodgkinson, to whom we are almost wholly indebted for our knowledge of the strength of pillars and struts, divides them into three classes:

1. Short pillars, whose lengths, compared with their diameters, is so short that they yield by crushing alone.
2. Medium, or short flexible pillars, whose lengths, compared with their diameters, is such that they fail partly by flexure or bending, and partly by crushing.
3. Long flexible pillars, whose lengths are so great, compared with their diameters, that they fail by flexure or bending, like a girder subject to a transverse strain, the breaking weight being much less than the crushing weight.

237. Formulæ for the Strength of Pillars Empirical.—The formulæ for the resistance of long pillars to flexure or bending are generally based upon the formulæ for the resistance of a girder supported at both ends and subject to a transverse weight or pressure, although the latter are not altogether satisfactory, nor do they entirely conform to the results of experiment.

The girder and the long pillar both yield from flexure, but there is a difference in the conditions of the two which will materially affect the formulæ for the resistance of the latter to the flexure derived from their resemblance. A girder, supported at both ends, and subject to a transverse pressure or weight, has its greatest longitudinal strain concentrated at one point, the compression on one side being equal to the tension on the opposite, and has no longitudinal strain at the ends; but a loaded pillar or strut must sustain, in some part of any transverse section, throughout its whole length, an amount of compression equal to the weight imposed upon it. If the line of this weight coincide with the axis of the pillar, failure will result from crushing, unaffected by the length, and there is no resemblance to the girder; and if, as is generally true of long pillars, this coincidence does not exist, the conditions of a girder are not immediately fulfilled, for the line of pressure may be nearer one side, so that the compression upon that side may be greater than upon the other, where the compression may be slight, or where there may actually be no strain. This may be true even with a slight bending of the pillar. But flexure soon produces tension in the convex side of the pillar, and this tension may resemble that in a loaded girder; that is, it may be greatest at one point and diminish in a certain ratio from that point towards the ends. It must diminish or vanish at some point, because it cannot exist at the ends, and it cannot vanish without neutralizing, or being neutralized by, an equal amount of compression; hence it either lessens the compression in the concave side, or when tension exists in a pillar, there is present an amount of compression equal to the weight imposed and to the tension. Therefore it would seem that a long pillar, yielding by flexure, would more nearly resemble a girder bending under a uniform load, and subject to two equal and opposite thrusts at its ends.

In the absence of sufficient experiment our only recourse is to such formulæ, and their modifications or adaptations to different forms, empirical though they be, as have generally been adopted with safety, using such as are adapted to truss struts and chord sections, each section being considered as a separate pillar.

238. Short Wooden Pillars.—The crushing weight of American yellow pine is given by Prof. Rankine as 5,400 pounds per square inch, and of oak as 6,700. These quantities seem small, but there is no doubt that the crushing weight of white pine, given by some American authors as 10,000 pounds per square inch, is too great. The extreme safe working load for wooden pillars may be put at 800 pounds per square inch for permanent structures; for temporary purposes this load may be exceeded. Rondolet found that wooden pillars do not yield by flexure until their length exceeds ten times their smallest diameter. The load of 800 pounds per square inch, however, may be carried until the length so exceeds the diameter that the formula for flexure, given beyond, gives a less weight, which is when the length is about $29\frac{1}{2}$ times the diameter.

239. Long Wooden Pillars.—Prof. Rankine gives this formula for the working strength of long rectangular wooden pillars:

$$W = \frac{300{,}000\, bd^3}{l^2}, \quad - \quad - \quad - \quad - \qquad (371)$$

where W is the safe working strain in pounds,

b is the breadth in inches,

d is the depth, or least lateral dimension, in inches,

l is the length in inches.

Example.—What is the safe working strain for a wooden pillar 10 feet long, 4 inches deep, and 5 inches in breadth?

Here, $$W = \frac{300{,}000 \times 5 \times 4^3}{14400} = 6667 \text{ pounds.}$$

240. Square Wooden Pillars. — "Of rectangular wooden pillars, it was proved experimentally, that the pillar of greatest strength, where the length and quantity of material is the same, is a square."—*Hodgkinson.*

241. Solid Pillars of different Transverse Sections.— "It appears that the strength of (long) circular, square, and triangular pillars of the same quality, weight, and length vary as 55.299, 51.537, and 61.056, the last being the strongest."—*Hodgkinson.*

Or if the strength of a long, round pillar be 100, that of a square solid pillar will be 93, and that of a triangular solid pillar 110, the pillars being of equal length and transverse section.

241.—The crushing strain of cast-iron is 85,000 pounds per square inch.

242. The Load for Cast-Iron Solid Pillars.—The formula for solid round cast-iron pillars, whose lengths exceeds 30 times their diameters, deduced from Hodgkinson, is

$$W = \frac{6500\, d^{3.5}}{l^{1.63}}, \qquad - \quad - \quad - \quad - \qquad (372)$$

in which W is the safe working load in pounds,

d, the diameter in inches,

l, the length in feet.

Tables of the powers of $d^{3.5}$ and $l^{1.63}$ are given at the end of the chapter.

Example.—What is the safe weight for a solid pillar of cast-iron 10 feet long and 3 inches in diameter? Taking the values of $10^{1.63}$ and $3^{3.5}$ from the tables, we have

$$W = \frac{6500 \times 46.765}{42.658} = 7078 \text{ pounds.}$$

243. The Load for Cast-Iron Hollow Pillars.

$$W = \frac{6500(d^{3.5} - d'^{3.5})}{l^{1.63}}, \qquad - \quad - \quad - \qquad (373)$$

in which W, d, and l have the same meaning as before, and d' is the diameter of the inside of the pillar in inches.

244. The Load for Cast-Iron Rectangular Pillars.— For square solid pillars,

$$W = \frac{9232\, d^{3.5}}{l^{1.63}}. \quad - \quad - \quad - \quad - \qquad (374)$$

For square hollow pillars,

$$W = \frac{9232(d^{3.5} - d'^{3.5})}{l^{1.63}}. \quad - \quad - \quad - \qquad (375)$$

For rectangular solid pillars,

$$W = \frac{9232\, bd^{2.5}}{l^{1.63}}. \quad - \quad - \quad - \quad - \qquad (376)$$

For rectangular hollow pillars,

$$W = \frac{9232(bd^{2.5} - b'd'^{2.5})}{l^{1.63}}, \quad - \quad - \qquad (377)$$

in which W and l have the same meaning as before; d is the outside of the square, or the least side of the rectangle in inches; d' the inside of the square, or least inner side of the rectangle in inches; b, the greater outer, and b' the greater inner dimensions of the rectangle.

Example.—What is the safe working load for a cast-iron rectangular pillar 15 feet long, 8×12 inches outside measurement, and with sides half an inch in thickness?

Here $b' = 11$, and $d' = 7$, whence, taking the powers from the tables,

$$W = \frac{9232\,(12 \times 181.02 - 11 \times 129.64)}{82.6093} = 83390.$$

245. The Load for Solid Triangular Cast Iron Pillar.—The formula for a solid cast-iron pillar of equilateral transverse section is

$$W = \frac{2291.4\, d^{3.5}}{l^{1.63}}, \quad - \quad - \quad - \quad - \qquad (378)$$

d being the side in inches.

246. The Safe Working Load for Cast Iron Pillars.—In the preceding formulæ for cast-iron the safe working load has been put at one-fifth of the breaking weight. Mr. Francis has put it at one-fifth, Navier at one-fifth, while Mr. Stoney puts it at one-fifth where there is no vibration, one-sixth where there is a moderate vibration, and one-tenth where the pillar is subject to a heavy jar.

247. Cast Iron Pillars of other Forms.—A cast-iron pillar of the + form is very weak to bear a strain, less than one-half of the strength of a hollow cylindrical pillar of equal weight and length; a pillar of the ├─┤ form is stronger than the preceding, but weaker than the hollow cylindrical. The proportions are as follows, the weights and lengths being equal:

Hollow Round, - - - -	1000
+ form, - - - - - -	443
├─┤ form, - - - - - -	746

248. Equality in the Thickness of Hollow Pillars not Important.—Mr. Hodgkinson remarks that, "where there is an inequality in the thickness of hollow pillars it does not produce much diminution of strength." Mr. Stoney adds: "In practice, neither the excess nor the want of thickness should exceed twenty-five per cent. of the average thickness. If, for instance, a hollow pillar is specified to be one inch in thickness, then in no place should the metal be less than three-quarters of an inch, nor more than one and a quarter inch thick.

249.—"In all the pillars with rounded ends, those with increased middles were stronger than uniform pillars of the same weight, the increase being about one-seventh of the weight borne by the former."—*Hodgkinson.* They were solid cast-iron pillars.

250. Formulæ for Medium Pillars.—The above formulæ apply to all pillars whose lengths exceed thirty times their external diameters (for cast-iron and timber, and sixty diameters for wrought iron). For pillars shorter than this they give too great a weight, and for such Mr. Hodgkinson has modified his formulæ, from which we have,

$$W' = \frac{5\,W\,c}{60\,W + 3c}, \qquad (379)$$

where W′ = the safe working load in pounds,

W = the safe working load in pounds, derived from the formulæ given above for long pillars,

c = the crushing weight of the pillar; that is, the crushing weight per square inch of the material, multiplied by the sectional area in inches.

251.—In hollow cast-iron pillars Mr. Hodgkinson found that no additional strength was obtained by enlarging the diameter at the middle. Solid square pillars "do not break in a direction parallel to their sides, but to their diagonals nearly."—*Hodgkinson.*

252. Wrought Iron.—Mr. Hodgkinson gives the crushing weight of wrought iron as 35,800 pounds per square inch, being much less than that of cast-iron; hence a short cast-iron pillar can resist a much greater crushing strain than wrought-iron. But, as Mr. Stoney remarks, the strength of very long pillars depends not on the strength of the material, but on its stiffness and capability of resisting flexure; hence, "although a short pillar of cast-iron will bear a much greater weight than a similar pillar of wrought-iron, yet a very long wrought-iron pillar will support a greater weight than a similar one of cast-iron, as the co-efficiency of elasticity is considerably higher than that of cast-iron."

253. Long Solid Wrought Iron Pillars.—The formula for the safe working load of solid wrought-iron pillars is,

$$W = \frac{24967\, d^{3.5}}{l^2}. \qquad (380)$$

The working load being put at one-fourth the breaking weight; W, *d*, and *l* having the same meaning as before.

254. Hollow Wrought Iron Pillars.—The strength per square inch of hollow wrought-iron pillars seem to depend greatly upon a ratio between the thickness of the plate and the diameter of the pillar, though this ratio is not satisfactorily known. Prof. Rankine says: "The *ultimate* resistance of a square wrought-iron pillar, when the thickness of the plate is not less than one-thirtieth the diameter or side of the pillar, is 27,000 pounds per square inch." The strongest form of a hollow rectangular pillar is where the chief part of the material is concentrated at the angles.

255. Relative Strength of Long Pillars.—Putting the strength of a long cast-iron at 1,000, the strength of similar pillars of other materials was found by Mr. Hodgkinson to be:

Cast-iron, - - - - - -	1,000
Wrought-iron, - - - - -	1,745
Cast-steel, - - - - -	2,518
Oak, - - - - - - -	109
Pine, - - - - - -	79

Hence, from the formulæ for cast-iron, the strength of similar pillars of other materials may be found.

II.—TENSION.

256.—No formula is required for tension, since the strength of any member of a truss subject to a tensile strain varies directly as its weakest transverse section, and is unaffected by the length; hence we need only to know the tearing weight of different materials and the safe working load.

The safe working tension for timber may be put at 900 pounds per square inch of the smallest transverse section.

Cast-iron is not suited for tension; its tearing weight is 15,680 pounds per square inch, and its safe working load should not exceed one-sixth of this.

The tensile strength of wrought-iron is 54,000 pounds per square inch; its safe working load should not exceed one-fourth of this.

Table I.—Powers of Lengths, or $l^{1.63}$.

$1^{1.63} = 1.$	$9^{1.63} = 32.927$	$17^{1.63} = 101.305$
$2^{1.63} = 3.095$	$10^{1.63} = 42.658$	$18^{1.63} = 111.197$
$3^{1.63} = 5.994$	$11^{1.63} = 49.828$	$19^{1.63} = 121.442$
$4^{1.63} = 9.58$	$12^{1.63} = 57.42$	$20^{1.63} = 132.032$
$5^{1.63} = 13.782$	$13^{1.63} = 65.423$	$21^{1.63} = 142.961$
$6^{1.63} = 18.552$	$14^{1.63} = 73.823$	$22^{1.63} = 154.223$
$7^{1.63} = 23.851$	$15^{1.63} = 82.609$	$23^{1.63} = 165.812$
$8^{1.63} = 29.651$	$16^{1.63} = 91.773$	$24^{1.63} = 177.723$

Table II.—Powers of Diameters, or $d^{3.5}$.

$1^{3.5} = 1.$	$4.75^{3.5} = 233.58$	$8.5^{3.5} = 1790.47$
$1.25^{3.5} = 2.184$	$5^{3.5} = 279.51$	$8.75^{3.5} = 1981.66$
$1.5^{3.5} = 4.134$	$5.25^{3.5} = 331.56$	$9^{3.5} = 2187.00$
$1.75^{3.5} = 7.09$	$5.5^{3.5} = 390.18$	$9.25^{3.5} = 2407.11$
$2^{3.5} = 11.314$	$5.75^{3.5} = 455.87$	$9.5^{3.5} = 2642.61$
$2.25^{3.5} = 17.086$	$6^{3.5} = 529.09$	$9.75^{3.5} = 2894.12$
$2.5^{3.5} = 24.705$	$6.25^{3.5} = 610.35$	$10^{3.5} = 3162.28$
$2.75^{3.5} = 34.488$	$6.5^{3.5} = 700.16$	$10.25^{3.5} = 3447.73$
$3^{3.5} = 46.765$	$6.75^{3.5} = 799.03$	$10.5^{3.5} = 3751.13$
$3.25^{3.5} = 61.886$	$7^{3.5} = 907.49$	$10.75^{3.5} = 4073.14$
$3.5^{3.5} = 80.212$	$7.25^{3.5} = 1026.08$	$11^{3.5} = 4414.43$
$3.75^{3.5} = 102.12$	$7.5^{3.5} = 1155.35$	$11.25^{3.5} = 4775.66$
$4^{3.5} = 128.00$	$7.75^{3.5} = 1295.85$	$11.5^{3.5} = 5157.54$
$4.25^{3.5} = 158.26$	$8^{3.5} = 1448.15$	$11.75^{3.5} = 5560.74$
$4.5^{3.5} = 193.31$	$8.25^{3.5} = 1612.83$	$12^{3.5} = 5985.96$

TABLE III.—POWERS OF DIAMETERS, OR $d'^{2.5}$.

$1^{2.5} = 1$	$4.75^{2.5} = 49.174$	$8.5^{2.5} = 210.63$
$1.25^{2.5} = 1.747$	$5^{2.5} = 54.90$	$8.75^{2.5} = 226.47$
$1.5^{2.5} = 2.756$	$5.25^{2.5} = 63.153$	$9^{2.5} = 243.00$
$1.75^{2.5} = 4.052$	$5.5^{2.5} = 70.943$	$9.25^{2.5} = 260.23$
$2^{2.5} = 5.657$	$5.75^{2.5} = 79.281$	$9.5^{2.5} = 278.17$
$2.25^{2.5} = 7.594$	$6^{2.5} = 88.181$	$9.75^{2.5} = 296.83$
$2.5^{2.5} = 9.882$	$6.25^{2.5} = 97.654$	$10^{2.5} = 316.23$
$2.75^{2.5} = 12.541$	$6.5^{2.5} = 107.716$	$10.25^{2.5} = 336.36$
$3^{2.5} = 15.588$	$6.75^{2.5} = 118.375$	$10.5^{2.5} = 357.25$
$3.25^{2.5} = 19.042$	$7^{2.5} = 129.642$	$10.75^{2.5} = 378.99$
$3.5^{2.5} = 22.917$	$7.25^{2.5} = 141.53$	$11^{2.5} = 401.31$
$3.75^{2.5} = 27.232$	$7.5^{2.5} = 154.05$	$11.25^{2.5} = 424.50$
$4^{2.5} = 32.00$	$7.75^{2.5} = 167.21$	$11.5^{2.5} = 448.48$
$4.25^{2.5} = 37.237$	$8^{2.5} = 181.02$	$11.75^{2.5} = 473.26$
$4.5^{2.5} = 42.967$	$8.25^{2.5} = 195.37$	$12^{2.5} = 498.83$

THE END.

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